Middlemen and Liquidity Provision*

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Abstract

We develop a monetary model in which a middleman, who intermediates between suppliers and consumers in retail markets, simultaneously operates as a financier to her suppliers, pooling funds to provide liquidity support for urgent needs. We demonstrate that, the middleman's advantage in retail technologies affects her role as a financier where she selects suppliers not only based on their profitability but also on their contributions to the overall liquidity pool. Somewhat surprisingly, costly liquidity from the money market (i.e. non-zero nominal interest rates) induces the middleman to create a liquidity pool that enables welfare-improving liquidity cross-subsidization among her suppliers.

Keywords: *Middlemen*, *Liquidity Pooling*, *Liquidity Cross-subsidization*, *Money and Credit*

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1 Introduction

Middlemen in retail markets (e.g. retailers, merchants, trading companies) often provide liquidity to their suppliers. Historically, middlemen and liquidity provision were closely related. During the colonial era, European trading companies like the Dutch East India Company offered merchant credit to local producers in Africa, Asia, and the Americas through advanced payments for their purchased goods such as spices, cotton, and tobacco. Similarly, in the 19th and early 20th centuries in the Southern United States, farmers without access to traditional credit could receive production inputs and capital (e.g., seeds, fertilizers, plows, livestock feed, etc.) in advance from merchants, using future crop yields as repayments.

Even to the day, when banking systems and money markets are established, the middlemen's role as a financier has not diminished. Rather, middlemen finance appears to become increasingly more important to the extent that a need emerges to support liquidity-constrained suppliers, just like in periods after the Covid crisis. This trend is also accelerated with the rise of Fintech which facilitates the enforcement of supplier credit, and middlemen become more able to identify a variety of liquidity needs and rescue the suppliers with deeper financial difficulties.[1]

Despite their significant impact in the industry, the link between middlemen and their liquidity provision has received little attention in the economics literature. To study the mechanism behind its cause and consequence, we develop a model which explicitly takes into account frictions in retail markets and liquidity constraints faced by heterogeneous suppliers. More precisely, we consider a retail market that opens over two sub-periods: early and late. Production can happen only in the early sub-period. Each supplier can trade either of the sub-periods, and which one he can enter is determined randomly. Suppliers do not have initial endowments but production requires costs c which must be paid early using money, e.g., by making cash payments to obtain raw materials. If a supplier is given a chance to trade early, then he obtains retail revenue early and uses it to cover production costs. In this case, production and trade can happen, just like in standard frictionless retail markets. If instead a supplier has to wait till late, then there is no way he can cover production costs and so he cannot produce nor trade. We denote by λ the probability of the latter event, which shall be referred to as a liquidity shock, to happen. Suppliers vary in c and λ and the pair (λ, c) captures supplier heterogeneity. We assume suppliers' type is publicly observable.

In this economy, we introduce a middleman who cannot produce goods but has the following retail and enforcement technologies. On the one hand, the middleman is able to better match with consumers, and it is represented by her lower probability of experiencing a liquidity shock, $m\lambda$ $(< \lambda)$, where the parameter $m < 1$ measures the middleman's efficiency in retail markets. This is

[1]Recently, major middlemen like Walmart, Amazon, Alibaba, and JD.com, as well as manufacturers such as GE, Nestlé, Siemens, and Samsung, have adopted this financing approach. Fintech service providers, including Taulia, C2FO, PrimeRevenue, and Tradeshift, supported this trend, enabling middlemen across various industries to offer the necessary liquidity to their chosen suppliers. See Section 5 for more details.

in the same spirit as Rubinstein and Wolinsky (1987) where the very reason of the emergence of middlemen is due to their relative advantage in matching efficiency over the original suppliers. On the other hand, the middleman has enforcement technologies for credit with suppliers. This finance service is costly but allows for *trade credit*, i.e., pooling unused money from some suppliers who obtain positive retail profits early, and *early payments*, i.e., allocating the pooled money to those who have to wait till late and so have no money to cover c.

Given her advantage in retail technologies, it is optimal for the middleman to offer retail intermediation service to all the available suppliers, i.e. selling products on their behalf using her better matching technologies. Hence, the decision of the middleman is boiled down to whether or not, and which of her suppliers, to offer the costly finance service described above. For this to be feasible, the middleman must ensure that the total liquidity she has in hand, including her own balance and the trade credit obtained from early-trading suppliers, is enough to meet the early payment obligations to the participating suppliers. Therefore, as for whether or not to provide the middleman-finance service to a given supplier, the middleman cares not only about how profitable this supplier is, but also about how much he can contribute to the liquidity pool.

We show that suppliers with higher λ's (who appreciate the liquidity support more) and lower c's (who have a higher profit margin) are more likely to make a positive contribution in profits. In contrast, suppliers with lower λ's (who are less likely to request early payment) and lower c's (who need a smaller amount of liquidity) are more likely to make a positive contribution in liquidity. The middleman optimally adopts *profit-based liquidity cross-subsidization*, where she uses trade credit from less profitable suppliers to support those who are more profitable but are in need of liquidity. This proves more profitable than selecting suppliers solely based on profits.

The significance of liquidity cross-subsidization is determined by the shadow value of liquidity for the middleman, which equals the multiplier of the middleman's liquidity constraint. The shadow value increases as the liquidity constraint becomes more stringent. At the optimal (interior) liquidity holding, the shadow value of liquidity equals the liquidity price in the market, i.e., the nominal interest rate. With a higher nominal interest rate, the middleman emphasizes the liquidity contribution more and the profit contribution less in selecting suppliers. Thus, she avoids using her own funds and, instead, relies solely on trade credit in high-interest-rate environments.

The provision of finance services is influenced by the middleman's matching advantage m. We show that, in the scenario where the middleman's liquidity holding and finance provision take interior solutions, as m decreases (i.e., the middleman becomes more efficient), the risk of liquidity shocks, and the benefit of middleman finance, are reduced. This induces the middleman to exclude all the marginal suppliers, and leads to a smaller scope of financed suppliers.

In our economy, the middleman's liquidity provision improves welfare, since it allows suppliers who experience a liquidity shock to continue their production. At the Friedman rule (where the nominal interest rate is zero), liquidity is not a concern for the selection of suppliers, and so the middleman only invites those with positive profits. At a positive nominal interest rate, outside liquidity becomes costly and so the middleman must compare the non-zero cost of her own liquidity holding (i.e., the positive nominal interest rate) versus the cost of using the liquidity contribution by participating suppliers. When the middleman is more efficient (with a smaller m), she can reduce the likelihood of her suppliers' liquidity shocks by more and so more suppliers are potentially available as liquidity contributors. Therefore, with a sufficiently efficient middleman, more suppliers are invited to the middleman's finance program as the nominal interest rate becomes positive. Then, the aggregate trading volume becomes higher, and the suboptimality of the Friedman rule follows. In other words, welfare-improving liquidity cross-subsidization is adopted by the middleman only when outside liquidity from the money market becomes costly because she does not have an incentive to do so at the Friedman rule.

In Section 4, we show that middleman finance can be active even when suppliers' access to the money market is allowed. For low nominal interest rates, suppliers with positive profit contributions choose to hold onto their money, rather than using the liquidity support of middleman, to prepare for the liquidity shock. For high nominal interest rates, consumers trade only with those suppliers with low prices, namely low c's, to avoid inflation costs. In either case, the middleman finance becomes unprofitable and so ceases to be active. For intermediate nominal interest rates, the middleman-provided liquidity and suppliers' own money holdings coexist. Specifically, suppliers with high λ and low c choose to hold money on their own, while a subset of the remaining suppliers opt for middleman finance.

In Section 5, we offer anecdotal evidence that supports the implications of our model and demonstrate its relevance to a number of financial arrangements, such as supplier finance, keiretsu in Japan and rural credit cooperatives in 19th-century Germany. All proofs are included in the Appendix. The rest of this section is a literature review.

Related Literature

It is well-established that middlemen can help mitigate search frictions by maintaining a strong market presence (Rubinstein and Wolinsky, 1987; Nosal, Wong and Wright, 2015), holding larger inventories (Watanabe, 2010, 2018, 2020; Li, Murry, Tian and Zhou, 2024), offering a broader range of commodities (Camera, 2001; Johri and Leach, 2002; Shevchenko, 2004; Smith, 2004; Dong, 2010), or providing quality assurance by leveraging their information expertise (Biglaiser, 1993; Li, 1998, 1999). Manufacturers can also act as middlemen since they could access to retail technologies that are not readily available to consumers and suppliers (Spulber, 1996). However, the role of middlemen to provide liquidity has not been studied in this literature.[2]

[2]Our paper is also related to the burgeoning literature on the hybrid or dual-mode of platform economies, e.g. Tirole and Bisceglia (2023), Madsen and Vellodi (2023), Gautier, Hu and Watanabe (2023), Etro (2023), Etro (2024), Shopova (2023), Hagiu, Teh and Wright (2022), Anderson and Bedre-Defolie (2022), Kang and Muir (2022), Padilla, Perkins and Piccolo (2022), Zennyo (2022), Etro (2021a), and Etro (2021b). However, these papers focus on platforms that act as

Among the New Monetarist models based on Lagos and Wright (2005), our paper is broadly related to the banking models, e.g., Berentsen, Camera and Waller (2007), Gu, Mattesini, Monnet and Wright (2013), and Andolfatto, Berentsen and Martin (2019), and the financial intermediation models, e.g. Bethune, Sultanum and Trachter (2022). A distinct feature of our model is the ex-ante selection of heterogeneous depositors (suppliers in our model). This feature is also absent in the nonmonetary banking literature following Diamond and Dybvig (1983). Unlike demand deposits, the middleman in our framework promises to advance a limited amount of liquidity to suppliers. Thus, runs can be avoided. Another difference would be that in this literature, financial inclusion is inconsequential but in our model, it is an essential ingredient.

Closely related to ours is the growing literature on money and corporate finance.[3] Rocheteau, Wright and Zhang (2018) emphasize the strategic role of firms' liquidity holdings. A lower nominal interest rate prompts firms to hold more cash, which helps to negotiate a favorable loan term with the bank. Bethune, Rocheteau, Wong and Zhang (2021) highlight a monetary channel through which a lower nominal interest rate decreases the banks' incentive to create lending relationships with firms who have a stronger bargaining position. Our model uncovers a novel channel of monetary policy transmission to corporate finance — the provision of trade credit. In the context of middleman-provided finance, a lower nominal interest rate induces the middleman to use her own money holdings more, and use trade credit of suppliers less. Thus, suppliers of higher profits, rather than higher liquidity, are more likely to be invited by the middleman.

In the literature on the coexistence of money and credit, Gu, Mattesini and Wright (2016) show that changes in credit limit have no impact on allocations and welfare. In their model, this occurs because money and credit are perfect substitutes and so real money balance can adjust perfectly to changes in credit conditions. Trade credit in our model is also a perfect substitute for money. However, since our credit is very different from theirs, it is not clear how to define the credit limit that is comparable to their model. One key difference would be that in our model, an extension of the size of intermediation always improves welfare because it makes trade/consumption happen even when suppliers are hit by a liquidity shock.

Finally, our model setup is closely related to Rhodes, Watanabe and Zhou (2021) who study the product assortment problem of a multi-product middleman. They show that the middleman's problem can be described as the choice of a set of points in a simple two–dimensional

intermediaries between consumers and third-party sellers while also offering their own first-party products.

[3]There is a literature on supply chain finance in operations management, e.g., Tunca and Zhu (2017), Devalkar and Krishnan (2019), Kouvelis and Xu (2021), with the focus on comparing supply chain finance with other types of financial arrangements for various stakeholders in the supply chain. Our paper differs from the literature in many major aspects, e.g., we study middlemen-provided finance as a contract between a middleman and multiple small suppliers rather than a single contract between one buyer firm and one supplier. Our study is also related to the trade credit literature in finance, e.g., Petersen and Rajan (1997), Burkart and Ellingsen (2004), Cuñat (2007), Giannetti, Burkart and Ellingsen (2011), Garcia-Appendini and Montoriol-Garriga (2013), Nocke and Thanassoulis (2014), and Bottazzi, Gopalakrishna and Tebaldi (2023), etc. This literature argues that suppliers have a monitoring advantage over banks, which motivates the provision of trade credit despite high implicit interest rates. We consider middleman finance to be a type of financing that enables early payment to suppliers based on the trade credit they provide, which eventually leads to liquidity reallocation among suppliers.

statistic, just like ours. The middleman's optimal product assortment includes high–value products with low profitability, which make a direct loss to the middleman, and low–value products with high profitability, which recoups those losses. We show that this mechanism can create the liquidity cross–subsidization that the middleman optimally induces when selecting among heterogeneous suppliers. Further, we endogenize the middleman's liquidity–holding decision, and link it to the extent to which liquidity cross-subsidization occurs in a standard monetary equilibrium.

2 The model

Our model builds on a monetary model of Lagos and Wright (2005). Time is discrete and continues forever. Each period consists of two subperiods: day and night. A decentralized market (DM) is open during the day for perishable indivisible goods. A centralized market (CM) is open during the night. In the CM, all agents can consume and produce a divisible general good with a price normalized to one. The general good is produced one-to-one using labor h. There exists another divisible good, called fiat money, that can be used as a medium of exchange. The fiat money can be traded for the general good in the CM at price ϕ per unit.

Agents. There are three types of agents: a mass one of consumers, a mass one of suppliers (*he*), and *one* middleman (*she*). In the DM, each supplier produces a unique, perishable, and indivisible good at a constant marginal cost c. Suppliers differ in $c \in [\underline{c}, \bar{c}]$ with $\bar{c} > \underline{c} > 0$, and c is publicly observable. Consumers are homogeneous and have unit demand for each good with a common utility $u \geq \bar{c}$. The middleman does not produce nor consume in the DM. Instead, she can buy goods from suppliers and resell them to consumers. Besides, she also has access to a costly finance technology that enables her to delay payments to suppliers and then make use of the funds to finance other suppliers that are in need of liquidity support. The details of the middleman's retail and finance technologies will be specified below.

In the CM, the utility function of consuming x units of the general goods, denoted by $U(x)$, is strictly increasing, concave, and twice continuously differentiable. We normalize $U(x^*) = x^*$ where x^* solves $U'(x^*) = 1$.

The middleman and the consumers live infinitely with a common discount factor $\beta \in (0, 1)$. In this section, we assume that suppliers only live for one period. That is, suppliers are new-born without holding any money at the beginning of the day. This assumption reflects the liquidity constraints faced by small enterprises in the real world. In Section 4, we will relax this assumption and allow suppliers to hold fiat money that can be used to meet their liquidity needs. No agents discount within a period.

Fiat money. Meetings in the DM are anonymous, and effort in the CM is non-contractible. Therefore, fiat money has the essential role as a medium of exchange. Consumers must prepare fiat money in the CM so as to trade during the day. The supply of fiat money is controlled by the government. Let M and M_{-1} be the money supply of the current and the previous periods, respectively, with $M = \gamma M_{-1}$ where γ is the growth rate of money. Changes in M occur during the night via lump-sum transfers to (taxes from) consumers if $\gamma > 1$ ($\gamma < 1$). Denote by T such a transfer (or a tax) is measured in terms of the general good. We focus our attention on a symmetric steady-state monetary equilibrium where agents of identical types choose identical strategies, and all real variables are constant over time. In particular, $\frac{\phi_{-1}}{\phi} = \gamma$. The nominal interest rate is given by the Fisher equation $1 + i = \gamma/\beta$, and we assume $\gamma > \beta$. The Friedman rule is the limiting case $i \to 0$.

Decentralized markets (DM). In the DM, trade takes place bilaterally. Without loss of generality, we assume that the trade surplus is split equally between the two parties. The equilibrium retail price is given by

$$p - c = \frac{u - c}{2}. \tag{1}$$

In the DM, suppliers need money to cover production costs c to produce a good. In the frictionless setups, this is not an issue because suppliers can do so using their retail revenue. However, suppliers' finance matters when there exists a disparity in the timing between production and trade, and a liquidity shock prevents them from receiving revenue before production.

To be more precise, suppose that a DM occurs in two sub-periods of the day, namely, *early* and *late* subperiods. We assume that production is possible only in the early subperiod. At the time of production, an idiosyncratic shock is realized, indicating whether a given supplier's good will match with consumers in the early or late subperiod.

We assume that with probability $1 - \lambda$, consumers match with the good early and thus they pay in the early subperiod. In this case, the supplier can use the retail revenue to cover the production costs. Production and trade occur even if the supplier does not hold money. In contrast, with probability λ, consumers match with the good late and thus they only pay in the late subperiod. In this case, since the payment happens in the future, the supplier's immediate need to cover c cannot be satisfied. Thus, the supplier faces a liquidity shock so that production and trade never occur. Essentially, the timing of matching with consumers, early or late, models the liquidity needs of suppliers. The late arrival of consumers is equivalent to a liquidity shock.

This setup captures real-world scenarios where suppliers' liquidity depends on their retail technologies. No trade occurs because of limited retail technologies possessed by suppliers to convince consumers to pay early rather than late. For instance,

- Display/advertisement: A supplier can display his good to consumers in the early subperiod with probability $1 - \lambda$ and in the late subperiod with probability λ. If consumers buy only after inspection, then it is only in the former case that the supplier can produce and trade. Better advertisement technologies increase the chance of early display.

- Delivery/inventory: A supplier can deliver his good to consumers in the early subperiod with probability $1 - \lambda$, and in the late subperiod with probability λ. If consumers pay only after delivery, then it is only in the former case that the supplier can produce and trade. Better inventory technologies increase the chance of early delivery.

- Production-to-order: A supplier has access to "production-to-order" technology with probability $1 - \lambda$ and can only "produce to inventory" with probability λ. Production-to-order allows suppliers to produce goods after receiving an order and payment from consumers. Then it is only when this technology is accessible that the supplier can produce and trade. Better promotion or communication with consumers, facilitated by competent sales persons, increases the chance of production to order.

We assume that the probability of a liquidity shock varies among suppliers and is publicly observable. Suppliers' ex-ante heterogeneity can be indexed by a pair (λ, c). Denote the two-dimensional space where (λ, c) belongs to by $\Omega \equiv [0, 1] \times [\underline{c}, \bar{c}]$ with $0 < \underline{c} < \bar{c} < u$. The pair (λ, c) follows a continuous distribution which has a cumulative distribution function G, and a density function g that is everywhere positive in Ω.

Finally, we assume that, due to a lack of enforcement technologies, there's no credit market among suppliers. Consequently, individual suppliers are unable to hedge against liquidity shocks or combine liquidity resources by themselves.

The retail technology. The middleman can sell goods on behalf of suppliers. We assume that the middleman has a relative matching advantage over all the original suppliers because she has better retail technologies (just like in Rubinstein and Wolinsky 1987). For example, the middleman has:

- better advertisement technologies that increase the chance of early display;
- better inventory technologies that increase the chance of early delivery;
- better promotion/communication technologies with consumers that increase the chance of production to order.

Thanks to these advanced retail technologies, when the middleman sells the good for a supplier with λ, the probability that the middleman matches the good with consumers in the early subperiod becomes $1 - m\lambda$ ($> 1 - \lambda$), where $m \in (0, 1)$ represents the middleman's matching advantage, i.e., the middleman faces a lower probability $m\lambda$ ($< \lambda$) of receiving a liquidity shock.

The finance technology. The middleman can also operate as a financier and offer liquidity to suppliers. The middleman has access to a *finance technology*, which allows her to, firstly, make credit deals with suppliers in the DM; and secondly, pool the retail revenue from suppliers for the purpose of supporting other suppliers who are in need of liquidity. The latter is possible due to the delayed payments to suppliers, essentially the trade credit from suppliers (more details below). We assume that using this finance technology is costly: For each financed supplier, there is a fixed cost (or disutility) k to the middleman where $k \in (0, \bar{k})$ with $\bar{k} \equiv \frac{(u-\underline{c})^2}{2(u+\underline{c})}$ (see footnote 5).

The contracts offered by the middleman. Observing (λ, c), the middleman offers a contract to each supplier. Given the matching advantage of middleman in the retail market, the middleman will choose to offer her intermediation service to every supplier. In particular, a pure *middleman contract* stipulates that: (1) The middleman sells the good on behalf of the supplier; (2) The middleman gives the supplier a reward $f_M(\lambda, c) \geq 0$ *immediately after receiving payment from consumers.* If no payment is received from consumers, the supplier is paid nothing.

Suppliers accepting a middleman contract can produce only if the middleman successfully matches the good with consumers early, which occurs with probability $1 - m\lambda$. This is the best the middleman can offer without finance technologies. Only then can the supplier use the revenue to cover production costs.

With finance technologies, the middleman can offer liquidity service together with the intermediation service. In particular, a middleman-finance contract (*hereafter, the finance contract*) stipulates that: (1) The middleman sells the good on behalf of the supplier; (2) The middleman gives the supplier a reward $f_F(\lambda, c) \geq 0$ *at the end of the period*; (3) The middleman pays the cost c to the supplier at the time of production (in the early subperiod).

The finance contract differs from the middleman contract in two aspects. First, in the finance contract, payments to suppliers are postponed to the end of the period. With the finance technology, this deferral allows the middleman to leverage the delayed payments as a liquidity source to fund suppliers that are in need of liquidity. Second, the finance contract extends liquidity support of c at the time of production, which ensures the supplier can produce and trade with probability one, even if the supplier does not have money in hand to cover production costs.

We assume that the middleman makes take-it-or-leave-it offers. Thus, suppliers' outside values matter. A supplier who does not accept a middleman's offer sells directly to consumers in the DM, in which case the supplier can produce and trade only if he is not hit by the shock (i.e. if he is matched with consumers early). Let $q(\lambda, c) \in \{0, 1\}$ be the selection function, where $q(\lambda, c) = 1$ if a (λ, c)-supplier is offered a finance contract, and $q(\lambda, c) = 0$ if he is offered a middleman contract.[4] With this, we can summarize the set of the middleman's offers by a triple:

$$\{q(\lambda,c), f_F(\lambda,c), f_M(\lambda,c)\}_{(\lambda,c)\in\Omega}.$$

Timing. The timing in the DM can be summarized as follows. First, the middleman observes (λ, c) for all suppliers, announces her offers, and selects which suppliers to invite and which contracts to offer. The selected suppliers then decide whether or not to accept the middleman's offer. Second, the liquidity shock is realized for each supplier and for each of the goods the middleman sells, and trade occurs in the DM. Suppliers under a finance contract can request early payment of c. Suppliers under a middleman contract receive $f_M(\cdot)$ immediately after consumers pay. Finally, the middleman settles any outstanding payments to the suppliers, specifically $f_F(\cdot)$, by the end of the period.

3 The monetary equilibrium

In what follows, we construct a symmetric steady-state monetary equilibrium. Let z be the real value of money holdings. We index consumers by superscript b, suppliers by superscript s, and the middleman by superscript m. We work backward and begin with the CM. At the beginning of night, a consumer who holds z^b money has an expected value $W^b(z^b)$ given by

$$W^b(z^b) = \max_{x,h,z^b_+} \{U(x) - h + \beta V^b_+(z^b_+)\},$$
$$\text{s.t. } x = z^b + T + h - \gamma z^b_+,$$

where $\gamma = \frac{\phi}{\phi_+}$ and V^b_+ denotes the expected value of entering into the next DM. Inserting the budget constraint and $U(x^*) = x^*$, we have

$$W^b(z^b) = z^b + T + \max_{z^b_+}\left\{-\gamma z^b_+ + \beta V^b_+(z^b_+)\right\}. \tag{2}$$

As standard in the literature, z^b_+ is determined independently of current money holding z^b.

Likewise, the middleman who holds a real value of $\tilde{L}$ fiat money has an expected value given by

$$W^m(\tilde{L}) = \max_{x,h,L_+} \{U(x) - h + \beta V^m_+(L_+)\}, \text{ s.t. } x = \tilde{L} + h - \gamma L_+,$$

where V^m_+ is the middleman's expected value of entering the next DM. Then,

$$W^m(L) = \tilde{L} + \max_{L_+}\left\{-\gamma L_+ + \beta V^m_+(L_+)\right\}. \tag{3}$$

[4] In equilibrium, active suppliers are either included in a middleman contract or a finance contract.

A supplier who holds z^s money entering the CM faces:

$$W^s(z^s) = \max_{x,h}\{U(x) - h\}, \text{ s.t. } x = z^s + h.$$

Since he lives only for one period, the supplier will use up all his money in the CM to purchase the general good, yielding $W^s(z^s) = z^s$. We will allow suppliers to hold money in the next section.

Consumers' money holdings and purchase decision. In the DM, consumers purchase indivisible goods from suppliers and the middleman using fiat money. Denote the set of available goods in the market by $\hat{\Omega} \subset \Omega$, which is realized after a liquidity shock happens but before consumers make a purchase decision. The production costs of goods in $\hat{\Omega}$ follow a distribution, and we denote the density by $\hat{g}(c)$. Since the middleman has a matching advantage over individual suppliers, as we will see, no supplier sells directly to consumers in equilibrium. As such, goods from suppliers that are in a middleman-finance contract ($q(\lambda, c) = 1$) are available to consumers with probability one, and goods from suppliers that are not financed ($q(\lambda, c) = 0$) are available to consumers with probability $1 - m\lambda$. We have

$$\hat{g}(c) = \int_0^1 \Big\{q(\lambda, c) + (1 - q(\lambda, c))(1 - m\lambda)\Big\} g(\lambda, c) d\lambda,$$

which is an object to be determined in equilibrium.

Let $\omega(c, i) \in \{0, 1\}$ be a consumers' purchase decision, where i is the nominal interest rate. $\omega(c, i) = 1$ means that consumers purchase the good, and $\omega(c, i) = 0$ means they do not. The dependence of ω on i will become clear shortly. The independence of ω on λ reflects the fact that retail prices p (see (1)) depends on c but not on λ. For this reason, let us write $p = p(c) = (u + c)/2$ to clarify the dependence of p on suppliers' type c.

A consumer who holds a real balance of z^b_+ has the DM value given by

$$V^b(z^b_+) = \max_{\omega(c,i)} \int_{\underline{c}}^{\bar{c}} [\omega(c, i)\ u] \hat{g}(c) dc + W^b\left(z^b_+ - \int_{\underline{c}}^{\bar{c}} [\omega(c, i) p(c)] \hat{g}(c) dc\right)$$

$$\text{s.t.} \quad \int_{\underline{c}}^{\bar{c}} [\omega(c, i) p(c)] \hat{g}(c) dc \leq z^b_+.$$

Consumers obtain a common utility u for each indivisible good and pay the price $p(c)$. Using $W^b(z^b_+) = z^b_+ + W^b(0)$, we have

$$V^b(z^b_+) = \max_{\omega(c,i)} \int_{\underline{c}}^{\bar{c}} \omega(c, i) \frac{u - c}{2} \hat{g}(c) dc + z^b_+ + W^b(0).$$

To solve the problem, we can set up the Lagrangian:

$$\mathcal{L} = \int_{\underline{c}}^{\bar{c}} \omega(c, i) \frac{u - c}{2} \hat{g}(c) dc + z^b_+ + W^b(0) + \mu^b \left(z^b_+ - \int_{\underline{c}}^{\bar{c}} [\omega(c, i) p(c)] \hat{g}(c) dc\right).$$

Denoting by $\mu^b \geq 0$ the Lagrange multiplier of the budget constraint, we get the optimal choice of consumption given by

$$\omega(c,i) = 1 \text{ iff } \frac{u-c}{2} - \mu^b p(c) \geq 0.$$

Note that $\mu^b \geq 0$ is determined by the consumer's choice of money holdings in the previous CM (see (2)):

$$\max_{z_+^b} -\gamma z_+^b + \beta V^b(z_+^b).$$

Applying $V^{b\prime}(z_+^b) = 1 + \mu^b$, the Euler equation, $\gamma = \beta V^{b\prime}(z_+^b)$, becomes

$$\frac{\gamma}{\beta} = 1 + \mu^b.$$

In steady state, the Fisher equation implies $\frac{\gamma}{\beta} = 1 + i$, and so $\mu^b = i$. This in turn implies that $\omega(c,i) = 1$ iff $\frac{u-c}{2} \geq ip(c)$. Inserting $p(c)$, the condition becomes $i \leq \frac{u-c}{u+c}$. Using this, we have that for goods in $\hat{\Omega}$:

$$\omega(c,i) = \begin{cases} 1 & \text{if } \frac{u-c}{u+c} \geq i, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

(4) shows that consumers' purchase decision depends on i. Define $i_1 \equiv \frac{u-\bar{c}}{u+\bar{c}}$ and $i_2 \equiv \frac{u-\underline{c}}{u+\underline{c}}$. Note that $i_2 > i_1 > 0$. For $i \leq i_1$, consumers hold enough money to buy all the available goods in the DM. As i increases above the critical value i_1, some goods become too costly for consumers to include in their consumption basket, given that their money holding becomes smaller. In other words, goods with costs

$$c > \bar{c}(i) \equiv \frac{1-i}{1+i}u$$

are not purchased by consumers and so these suppliers drop out of the market one by one as i increases. Eventually, when i reaches the critical value i_2, consumers cannot afford to buy any goods available in the retail market, and so for $i > i_2$ no suppliers can make a sale.

To summarize, an effective space of suppliers is defined as

$$\Omega(i) \equiv [0,1] \times [\underline{c}, \bar{c}(i)] \subset \Omega \tag{5}$$

where now $\bar{c}(i) = \bar{c}$ for $i \leq i_1$ and $\bar{c}'(i) < 0$ for $i \in (i_1, i_2)$. $\Omega(i)$ is nonempty for $i < i_2$. The consumers' money-holding imposes a constraint on the middleman's supplier selection problem as we see in the following.

Suppliers' participation decision. A newborn supplier indexed by (λ, c) has zero money holdings. If $c \geq \bar{c}(i)$, the supplier's DM value is zero. Consider a supplier with $c < \bar{c}(i)$, who has an option of selling directly in the DM and earns $W^s\Big((1-\lambda)(p-c)\Big)$. Noting that $W^s(z^s) = z^s$ and applying (1), we have $W^s\Big((1-\lambda)(p-c)\Big) = (1-\lambda)(u-c)/2$. Since the middleman can observe

(λ, c), she can make the rewards f_j dependent on (λ, c). To entice the supplier to participate, it is sufficient for the middleman to offer him the value of his outside option so that

$$f_F(\lambda, c) = \frac{(1-\lambda)(u-c)}{2}, \tag{6}$$

and

$$f_M(\lambda, c) = \frac{(1-\lambda)(u-c)/2}{1-m\lambda} + c. \tag{7}$$

f_M differs from f_F because, in a middleman contract, production costs c are covered by the supplier, not the middleman. The reward f_M is given to the supplier only when the middleman successfully trades, which happens with probability $1 - m\lambda$ rather than $1 - \lambda$. With these fees, all the active suppliers are induced to accept an offered contract.

3.1 The middleman's problem

In the CM, the middleman must decide how much money to carry to the DM and in the DM which suppliers to finance. Suppose the middleman holds a real balance of L from the CM. We follow backward induction and work on the supplier selection problem first.

3.1.1 The middleman's supplier selection problem

Suppose $i < i_2$ so that the set of suppliers $\Omega(i)$ is non-empty. Below, we first derive the profit and liquidity the middleman obtains from individual suppliers in the DM.

Under a middleman contract, the middleman's expected profit from a supplier (λ, c) is

$$\pi_M(\lambda, c) = (1-m\lambda)\left(p - f_M(\lambda, c)\right) = (1-m)\lambda(u-c)/2, \tag{8}$$

which is positive since $m < 1$. The second equality follows from (1) and (7). The source of the profit is the middleman's matching advantage: the supplier does not receive a liquidity shock (and can trade successfully by himself) with probability $1 - \lambda$, whereas the middleman can do so with probability $1 - m\lambda$; the difference is given by $1 - \lambda - (1 - m\lambda) = (1 - m)\lambda$. Note that without incurring the finance technology cost k, the middleman cannot enforce any credit deals that allows him to fund suppliers. Instead, she only provides intermediation service and consumes the earned profits in the CM.

If financed by the middleman, participating suppliers can produce even if matches occur in the late subperiod. Thus, under a finance contract, the middleman's expected profit from a supplier (λ, c) is

$$\pi_F(\lambda, c) = p - c - f_F(\lambda, c) - k = \lambda(u-c)/2 - k, \tag{9}$$

where the middleman receives payment p from consumers, covers the supplier's production costs c in the form of advanced payment, and rewards the supplier by f_F at the end of a period. The second equality follows from (1) and (6). The expected profit is higher with a higher λ (as the supplier is less likely to trade if he chooses to operate independently) and a lower c (as the good has a higher profit margin).

When providing the finance service, liquidity is actually an issue because the middleman needs to cover the production cost c for all the participating suppliers at the time of production. The source of this funding is the revenue p from early matches which occur with probability $1 - m\lambda$ for a supplier of type λ. Hence, the net expected amount of money that a supplier (λ, c) contributes to the middleman at the time of production is

$$\theta_F(\lambda, c) = (1 - m\lambda)p - c = (1 - m\lambda)(u + c)/2 - c. \tag{10}$$

Let Θ be the total liquidity contributed by all the suppliers who are financed by the middleman:

$$\Theta = \int_{\Omega(i)} \Big[q(\lambda, c)\theta_F(\lambda, c) \Big] dG.$$

Then the liquidity constraint that the middleman faces in the DM can be written as:

$$\Theta + L \geq 0. \tag{11}$$

The liquidity constraint states that the total liquidity contribution of financed suppliers, plus the available liquidity $L \geq 0$ that is held by the middleman herself should be non-negative.

Using (8) and (9), the middleman's value of entering the DM can be written as

$$V^m(L) = \max_{\{q(\lambda,c)\}_{(\lambda,c)\in\Omega(i)}} \left\{ W^m \left(L + \int_{\Omega(i)} \Big[(1 - q(\lambda, c))\pi_M(\lambda, c) + q(\lambda, c)\pi_F(\lambda, c) \Big] dG \right) \right\},$$

subject to the liquidity constraint (11). Using $W^m(z^m) = z^m + W^m(0)$, and defining $\Delta\pi$ as the incremental change in profits when a supplier (λ, c) is financed compared to not being financed:

$$\Delta\pi(\lambda, c) \equiv \pi_F(\lambda, c) - \pi_M(\lambda, c) = m\lambda(u - c)/2 - k,$$

the middleman's problem of selecting suppliers can be formulated as

$$\max_{\{q(\lambda,c)\}_{(\lambda,c)\in\Omega(i)}} \int_{\Omega(i)} [q(\lambda, c)\Delta\pi(\lambda, c)] \, dG,$$

subject to (11). The problem can be understood as the middleman obtaining $\pi_M(\lambda, c)$ for all the active suppliers and additionally deciding whether to finance suppliers to earn $\Delta\pi(\lambda, c)$ subject to (11).

The middleman's problem defined above is an optimization of functionals, and the optimal solution can be derived by using the following Lagrange method (see e.g., Rhodes, Watanabe and Zhou 2021). Let $\mu \geq 0$ be the multiplier associated with the liquidity constraint (11). We can construct the Lagrangian:

$$\mathcal{L} = \int_{\Omega(i)} \Big[q(\lambda, c) \Big(\Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \Big) \Big] dG(\lambda, c).$$

Note that $\Delta\pi(\lambda, c)$ and $\theta_F(\lambda, c)$ can be positive or negative depending on the parameters. In particular, given the cost k of using the finance technology, it is not profitable to fund all suppliers, i.e., there exist suppliers with negative $\Delta\pi(\cdot)$.

Using this Lagrangian, the solution to the middleman's problem can be obtained as an optimal selection policy that depends not only on (λ, c) but also on μ. With a slight abuse of notation, we shall refer to this optimal policy to finance a supplier as $q(\lambda, c, \mu)$, which is given by:

$$q(\lambda, c, \mu) = \begin{cases} 1 & \text{if } \Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \geq 0; \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

Condition (12) indicates that $q(\lambda, c, \mu) = 1$ consists of three possible scenarios:

$$\Delta\pi(\lambda, c) \geq 0,\ \theta_F(\lambda, c) \geq 0, \tag{13a}$$

$$\Delta\pi(\lambda, c) > 0,\ \theta_F(\lambda, c) < 0,\ -\Delta\pi/\theta_F \geq \mu, \tag{13b}$$

$$\Delta\pi(\lambda, c) < 0,\ \theta_F(\lambda, c) > 0,\ -\Delta\pi/\theta_F \leq \mu. \tag{13c}$$

In scenario (13a), the middleman selects suppliers with positive increments in profits $\Delta\pi$ and positive liquidity contribution θ_F to finance. In scenario (13b), the middleman selects suppliers with positive increments in profits $\Delta\pi$ and negative liquidity contribution θ_F to finance, provided the gross return of liquidity, measured by $-\Delta\pi/\theta_F$, is higher than the shadow value of liquidity μ. In the last scenario (13c), the middleman selects suppliers with negative $\Delta\pi$ and positive θ_F to finance, as these suppliers contribute to the aggregate liquidity of the middleman. The cost of getting one unit of liquidity from these suppliers is $-\Delta\pi/\theta_F$, and the middleman should absorb liquidity from these suppliers if $-\Delta\pi/\theta_F \leq \mu$.

To illustrate the three scenarios in a figure, we insert $\Delta\pi(\cdot)$ and $\theta_F(\cdot)$ and obtain three boundaries that lie in $\Omega(i)$:

$$\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\theta_F}(\lambda) \equiv \frac{1 - m\lambda}{1 + m\lambda} u, \tag{14a}$$

$$\Delta\pi(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\Delta\pi}(\lambda) \equiv u - \frac{2k}{m\lambda}, \tag{14b}$$

$$\Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq b(\lambda, \mu) \equiv \frac{m\lambda u - 2k + \mu(1 - m\lambda)u}{m\lambda + \mu(1 + m\lambda)}. \tag{14c}$$

Note that the right-hand side of (14c) is a "weighted average" of the right-hand sides of the first two. We plot these three boundaries in Figure 1, annotated by $\theta_F(\lambda, c) = 0$, $\Delta\pi(\lambda, c) = 0$, and $\Delta\pi + \mu\theta_F = 0$, respectively. The intersection is denoted by (λ_0, c_0). Any suppliers below

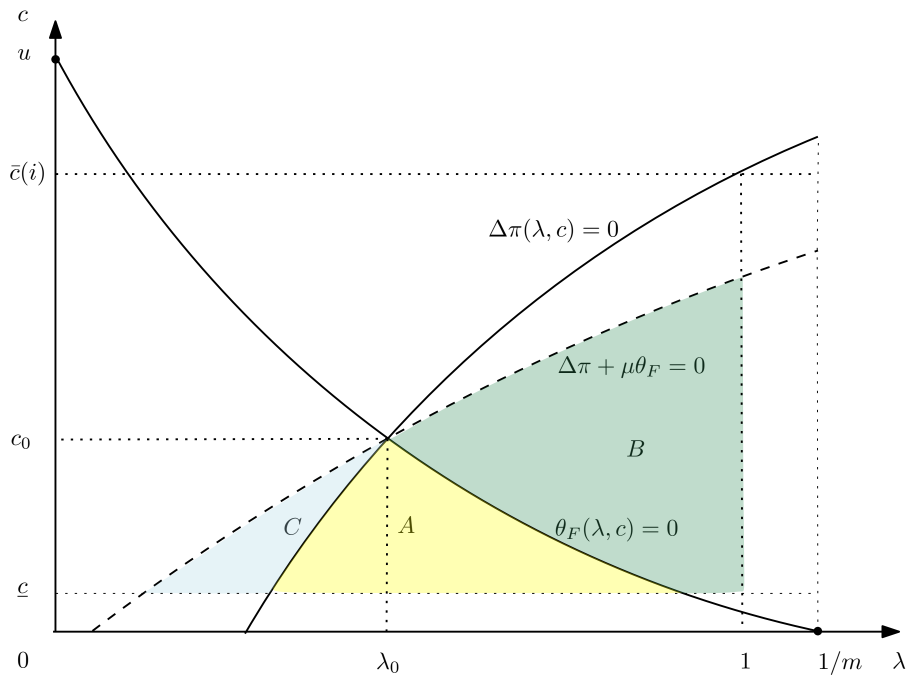


Figure 1: Middleman's selection of suppliers

$\theta_F(\lambda, c) = 0$ contribute to the liquidity pool, and any suppliers below $\Delta\pi(\lambda, c) = 0$ contribute to the middleman's profits.[5]

The overlapping region A represents suppliers in scenario (13a), which are financed by the middleman because they contribute to both profits $\Delta\pi$ and liquidity θ_F. Suppliers in region B, corresponding to scenario (13b), have net liquidity needs, $\theta_F < 0$, while contributing to profits $\Delta\pi > 0$. Suppliers in region C, corresponding to scenario (13c), when included in a finance contract, give the middleman lower profits $\Delta\pi < 0$, but contribute to the liquidity pool. Suppliers that are not in A, B or C are offered only intermediation service (but not finance service).

Overall, the middleman adopts what we call a *profit-based liquidity cross-subsidization* strategy. This involves using the positive net liquidity contributions from suppliers in regions A and C to address the liquidity needs of suppliers in region B. In particular, when the middleman uses liquidity contributions from region C, it incurs a cost in the form of reduced (or negative) profits to these suppliers. However, when providing liquidity support to suppliers in region B, the middleman expects a return in the form of positive profits from these suppliers.

In the standard liquidity pooling (a la Diamond and Dybvig 1983), agents are homogeneous and so, translated into our context, only those who make a positive profit contribution would be selected. We have demonstrated that with heterogeneous agents (each agent contributes profit and liquidity differently), this strategy is actually sub-optimal.

It remains to determine μ, the shadow value of liquidity for the middleman. If (11) is binding,

[5]It worth noting that $k < \bar{k}$ ensures that $c_0 > \underline{c}$. Also, Figure 1 is drawn with $\lambda_0 < 1$ and $\bar{c}(i) > c_0$. We will look into the cases where $\lambda_0 \geq 1$ or $\bar{c}(i) < c_0$ in later sections.

then μ is determined by

$$L = -\Theta(\mu, i) \equiv -\int_{\Omega(i)} q(\lambda, c, \mu)\theta_F(\lambda, c)dG. \tag{15}$$

Note that $\Theta(\mu, i)$ depends on i because the set of suppliers participating in the DM, $\Omega(i)$, depends on i. If (11) is slack, then $\mu = 0$. In this case, the middleman selects suppliers irrespective of liquidity concerns, i.e., the optimal selection policy is to select suppliers solely based on $\Delta\pi$.

Lemma 1 *If $\Theta(0, i) + L < 0$, then there exists a unique $\mu > 0$ that satisfies (15); and otherwise, $\mu = 0$.*

The liquidity value μ can be zero if the participating suppliers provide a sufficiently large amount of liquidity, $-\Theta(0, i) \leq L$. Note that this can be either with a positive liquidity pool $\Theta(0, i) \geq 0$ or a negative liquidity pool $\Theta(0, i) < 0$. Otherwise, the middleman's endowment has a positive liquidity value, $\mu > 0$.

For the finance contracts to be active, the set of suppliers with $\Delta\pi > 0$ need to be non-empty. A necessary and sufficient condition is $\Delta\pi(1, \underline{c}) > 0$, or equivalently, $\frac{k}{m} < \frac{u - \underline{c}}{2}$. This, along with $c_0 > \underline{c}$ (implied by $k < \bar{k}$), ensures that region A in Figure 1 always exists. Selecting suppliers in region A (those with both a positive $\Delta\pi$ and a positive θ_F) to finance is feasible with regard to the liquidity constraint and results in positive profits.

Lemma 2 *There exist suppliers that are financed by the middleman if and only if $k/m < (u - \underline{c})/2$.*

The intuition of this lemma is as follows. Whether the middleman finance is activated or not hinges on two parameters. First, the finance service requires a fixed cost k. When k is large, the incremental profit $\Delta\pi$ decreases, making it less attractive to finance suppliers. Second, when m is small, indicating that the middleman has a high matching advantage, consumers are likely to be matched in the early subperiod. This reduces the benefit of financing suppliers. In the extreme case, if $m \to 0$, the liquidity issue is eliminated altogether, making the finance contracts unnecessary. We summarize the results so far in the following theorem.

Theorem 1 (Selection of suppliers) *Taking $L \geq 0$ as given, and assuming $k/m < (u - \underline{c})/2$, the middleman's profit-maximizing strategy exists uniquely with the selection policies to finance suppliers $q(\lambda, c, \mu)$, satisfying (12), the reward to suppliers $f_F(\lambda, c)$ and $f_M(\lambda, c)$ satisfying (6) and (7), and the shadow value of liquidity $\mu \geq 0$ uniquely determined in Lemma 1.*

Corollary 1 *$\mu(L, i) > 0$ is strictly decreasing in L if $\Theta(0, i) + L < 0$.*

The middleman's available liquidity L shapes the feasibility of the middleman finance contracts via the liquidity constraint and especially μ. It is intuitive that μ is strictly decreasing in $L \in [0, -\Theta(0)]$: an additional unit of the middleman's money holding is appreciated more when her money holdings are relatively low. If L is higher, the curve $\Delta\pi + \mu\theta_F = 0$ is closer to $\Delta\pi = 0$, and the middleman selects suppliers primarily based on profits. If L is lower (which leads to a larger μ), $\Delta\pi + \mu\theta_F = 0$ is closer to $\theta_F = 0$. Then liquidity becomes more important when selecting suppliers, and the middleman relies more on liquidity cross-subsidization among suppliers. It is important to note that even a supplier that has high profits may not be chosen for the finance contract if he contributes little to the liquidity pool.

3.1.2 Middleman's money holdings

In the above analysis, the middleman's liquidity holding L is taken as exogenously given. We now derive it endogenously as stated in (3). The first order condition is

$$\phi_{-1} \geq \beta\phi V^{m\prime}(L/\phi),$$

with equality if and only if $L > 0$. Applying the Envelop condition $V^{m\prime}(L/\phi) = 1 + \mu$, we can write the first order condition as:

$$\phi_{-1} \geq \beta\phi(1+\mu).$$

Applying $\frac{\phi_{-1}}{\phi}\frac{1}{\beta} = \frac{\gamma}{\beta} = 1 + i$ in steady state, the first order condition can be simplified to

$$i \geq \mu. \tag{16}$$

This is essentially the Euler equation that determines the middleman's money holdings as a function of the nominal interest rate. Recall that $\mu(0,i)$ is the shadow value of liquidity to the middleman if her liquidity holding $L = 0$. From Lemma 1, we have that $\mu(0,i) > 0$ if and only if $\Theta(0,i) < 0$, and $\mu(0,i) = 0$ otherwise. Then we can characterize the middleman's optimal money holdings by comparing i and $\mu(0,i)$.

There are two scenarios to consider. In the first scenario, $\Theta(0,i) < 0$, which implies $\mu(0,i) > 0$ (see Corollary 1). If the nominal interest rate is relatively high, namely $i \geq \mu(0,i)$, the optimal money holding of the middleman is $L(i) = 0$, indicating that the funding source is entirely given by the pooled liquidity of suppliers. If the nominal interest rate is relatively low, namely $i < \mu(0,i)$, then (16) holds with equality, and the middleman holds a positive amount of money $L(i) = -\Theta(i,i) > 0$.

In the second scenario, $\Theta(0,i) > 0$, which implies $i > \mu(0,i) = 0$ (see Lemma 1), and the middleman can finance all the suppliers with positive profit contributions $\Delta\pi$, without holding money, $L(i) = 0$.

Lemma 3 (Middleman's money holdings) *The optimal money holdings of the middleman follow* $L(i) = -\Theta(i,i) > 0$ *if* $i < \mu(0,i)$, *and* $L(i) = 0$ *otherwise.*

To summarize, the value of liquidity with the middleman's optimal money holdings is given

[5] Note that $k < \bar{k}$ ensures that $c_0 > \underline{c}$.

by

$$\mu\left(i\right)=\mu\left(L(i),i\right)=\begin{cases}i & \text{if } i\leq\mu(0,i),\\ \mu(0,i) & \text{otherwise.}\end{cases} \tag{17}$$

where $L(i)$ is determined in Lemma 3.

3.2 The monetary equilibrium

For a monetary equilibrium to exist, consumers must hold a positive amount of money during the day, which requires $i < i_2$.

Theorem 2 *A monetary equilibrium exists if and only if $i \in (0, i_2)$, and if it exists it is unique, satisfying:*

- *the real balance of consumers is given by $z^b = \int_{\underline{c}}^{\bar{c}} \omega(c,i)p(c)\hat{g}(c)dc$, where $\omega(c,i)$ represents consumers' purchase decision as is given by (4);*
- *the set of effective suppliers is given by (5);*
- *the middleman operates with $\{q(\lambda, c), f_F(\lambda, c), f_M(\lambda, c), \mu(i), L\}$, as is characterized by Theorem 1, Lemma 3 and (17).*

The following corollary says that at the Friedman rule, the liquidity constraint is slack and so the middleman selects suppliers solely based on profit gains.

Corollary 2 *As $i \to 0$, it holds that $\mu(i) \to 0$.*

3.2.1 Changes in the nominal interest rate

Next, we examine how the nominal interest rate i, representing liquidity costs in our model, influences the shadow value of liquidity $\mu(i)$, the middleman's liquidity holdings $L(i)$, and the selection of suppliers to finance.

In general, there are potentially two effects. First, there is a *direct* effect when $\mu(i) = i$: the middleman's liquidity holding L decreases with i. In Figure 1, this is shown as the selection curves' $\Delta\pi + \mu\theta_F = 0$ clockwise rotation around (λ_0, c_0). Second, there is an *indirect* effect: a higher i increases the consumers' money-holding cost. That is, i can constrain the feasible set of suppliers via the upper bound $\bar{c}(i)$, which eventually affects the selection of suppliers to finance. The indirect effect of i has an ambiguous effect on $L(i)$ and $\mu(i)$, which is determined by how the pooled liquidity from suppliers changes.

Several cases are possible, depending on the level of i and the liquidity holding of the middleman. We start with the simple case of $i < i_1$, where $\Omega(i) = \Omega$ and $\mu(0,i)$ is independent of i. In this case, i only affects the middleman's selection via the *direct effect.* It follows immediately that whenever $i < \mu(0,i)$, the shadow value of liquidity $\mu(i) = i$ is strictly increasing in i, and $L(i) = -\Theta(\cdot)$ is positive and strictly decreasing in i.

When i surpasses i_1 and continues to increase, $\bar{c}(i)$ decreases, and eventually, it will intersect with the middleman's selection curve $b(\lambda, \mu(i))$ defined in (14c). At this point, the indirect effect of i manifests itself. Without loss of generality, we assume that $\bar{c} > c_0 > \underline{c}$ and $\lambda_0 < 1$ in the following analysis.[6]

There are two cases depending on whether $L > 0$, where $\mu(0,i) > i$ holds, or $L = 0$, where $\mu(0,i) \leq i$ holds. Consider first the scenario of $\mu(0,i) > i$, namely, $\mu(i) = i$ and $L(i) > 0$. Let i_0 represent the interest rate such that $\bar{c}(i_0) = c_0$:

$$i_0 = \left(k + \sqrt{k^2 + 4uk}\right) / (2u). \tag{18}$$

Given $\mu(i) = i$, the following lemma characterizes how $\bar{c}(i)$ restricts the feasible set of suppliers, contingent upon whether $i < i_0$ or not.

Lemma 4 *Suppose that $\mu(i) = i$. If $i < i_0$, then $b'_\lambda(\lambda, i) > 0$ and $b(\lambda, i)$ lies entirely below $\bar{c}(i)$. If $i > i_0$, then $b'_\lambda(\lambda, i) < 0$ and $b(\lambda, i)$ lies entirely above $\bar{c}(i)$.*

According to Lemma 4, when $i < i_0$, $b(\cdot)$ lies below $\bar{c}(i)$, meaning that all suppliers that are financed satisfy the condition $c \leq \bar{c}(i)$. This relationship is depicted in Figure 2(a). Notably, since $\bar{c}(i)$ has no influence on the middleman's optimal choice of financing suppliers, i impacts the middleman's liquidity holding solely through the direct effect. Therefore, like in the scenario $i \leq i_1$, here as well, $L(i)$ is strictly decreasing in i.[7]

When $i > i_0$, $b(\cdot)$ lies above $\bar{c}(i)$, meaning that all suppliers in $\Omega(i)$ are financed, see Figure 2(b). In other words, the pure middleman contract becomes inactive. In particular, in this case, $\bar{c}(i)$ rather than $b(\lambda, \mu)$, determines which suppliers to finance and how much money to hold. Since the liquidity constraint of the middleman is binding, $L(i)$ is given by [8]

$$L(i) = -\Theta(i,i) = -\int_{\Omega(i)} \theta_F(\lambda, c) dG = -\int_0^1 \int_{\underline{c}}^{\bar{c}(i)} \theta_F(\lambda, c) g(\lambda, c) dc d\lambda > 0.$$

A lower $\bar{c}(i)$ may increase or decrease the total liquidity from available suppliers. Thus, $L(i)$ can increase or decrease with i. For example, if the total liquidity decreases with i,

$$\frac{d\Theta(i,i)}{di} = \int_0^1 \bar{c}'(i) \theta_F(\lambda, \bar{c}(i)) g(\lambda, \bar{c}(i)) d\lambda < 0,$$

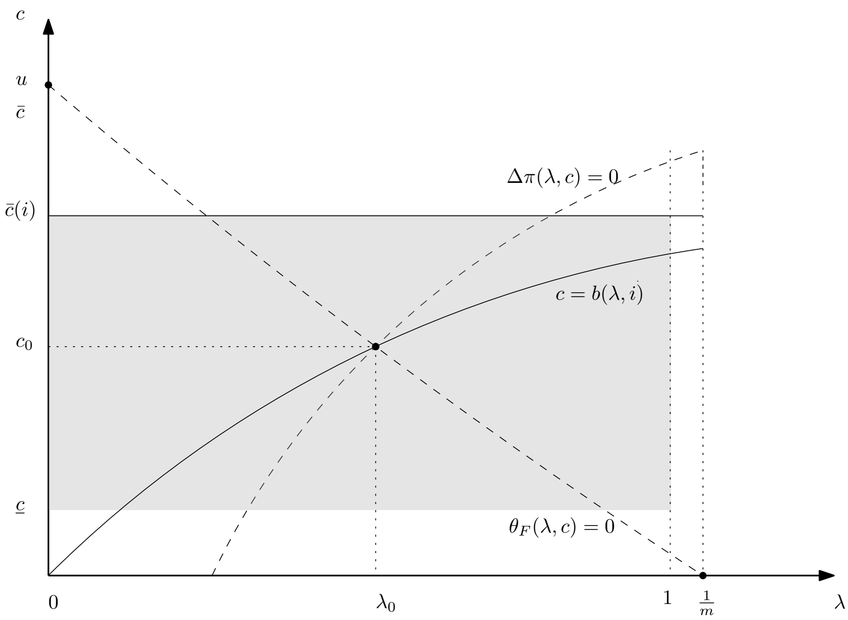
then $L(i)$ increases, despite of a higher cost of liquidity.

Consider the second scenario where $\mu(0,i) < i$, namely, $\mu(i) = \mu(0,i)$ and $L(i) = 0$. In this

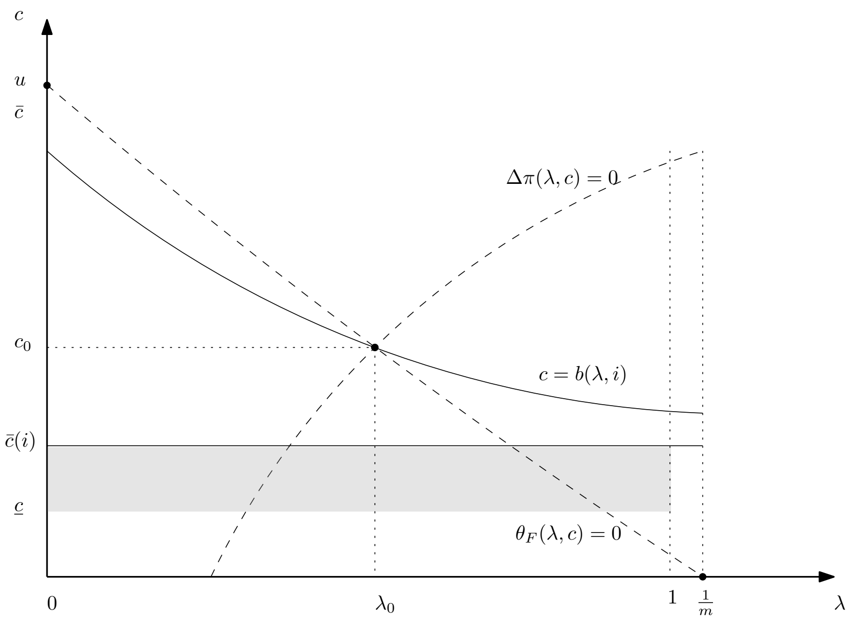
[6] $c_0 > \underline{c}$ is guaranteed by the assumption $k < \bar{k}$, and $\lambda_0 < 1$ is guaranteed by $m > \tilde{m}$ (see (19) below).

[7] At $i = i_0$, $c = b(\lambda, i_0)$ becomes a horizontal line, coinciding with $c = c_0$.

[8] In the scenario with $\mu(0,i) > i > i_0$, there exists a range of μ that selects the same set of suppliers as $\mu = i$, i.e., all suppliers with $c \leq \bar{c}(i)$ are selected. Thus, all such μ's give the same (negative) pooled liquidity from suppliers: $\Theta(\mu, i) = \Theta(i,i)$. Since all such μ's lead to the same allocation, we impose $\mu = i$, which is consistent with (17).

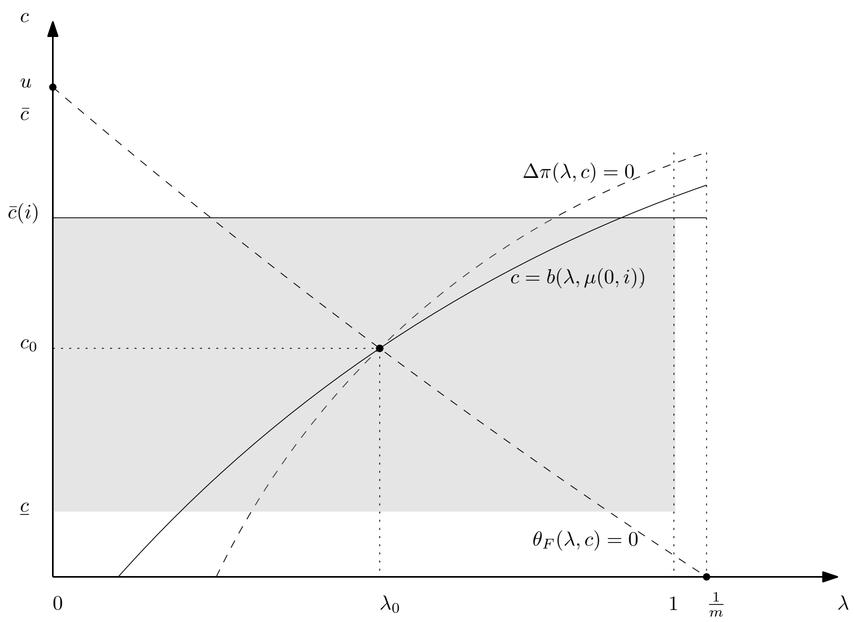


(a) $\mu(i) = i$, $b(\lambda, i)$ lies below $\bar{c}(i)$ when $i < i_0$

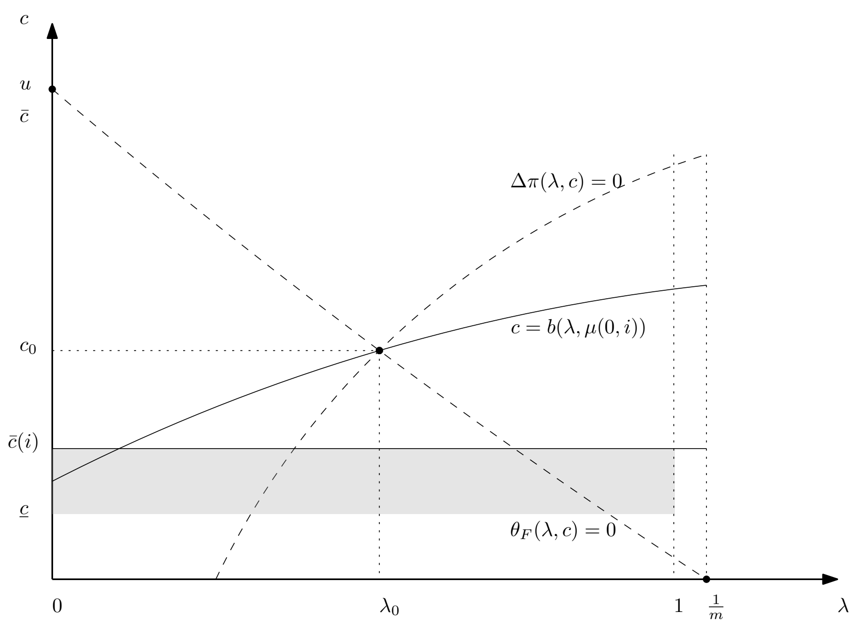


(b) $\mu(i) = i$, $b(\lambda, i)$ lies above $\bar{c}(i)$ when $i > i_0$

Figure 2: $b(\lambda, i)$ and $\bar{c}(i)$



(a) $\mu(i) = \mu(0, i)$, $b(\lambda, \mu(0, i))$ and $\bar{c}(i)$ intersect when $i < i_0$



(b) $\mu(i) = \mu(0, i)$, $b(\lambda, i)$ and $\bar{c}(i)$ intersect when $i > i_0$

Figure 3: $b(\lambda, \mu(0, i))$ and $\bar{c}(i)$

case, Lemma 4 does not apply, and i affects the decision of financing suppliers only through the *indirect effect* of $c \leq \bar{c}(i)$. If $i < i_0$, which is illustrated in Figure 3(a), then as i increases, $\bar{c}(i)$ crosses $c = b(\cdot)$, and excludes suppliers of negative liquidity contributions. Thus, the total liquidity from suppliers would increase. As a result, $\mu(0,i)$ must decrease (provided it is positive). That is, the middleman relies less on the liquidity cross-subsidization among suppliers.

If $i > i_0$, the shadow value liquidity $\mu(0,i)$ may increase or decrease in i (see the illustration of Figure 3(b)). $\mu(0,i)$ increases in i if a lower $\bar{c}(i)$ reduces the pooled liquidity from suppliers, namely, $\partial\Theta(\mu,i)/\partial i < 0$. Then, $\mu(0,i)$ must increase, leading to more liquidity cross-subsidization among suppliers.[9] We summarize these results in the following proposition.

Proposition 1 *Suppose $\mu(0,i) > i$, then $\mu(i) = i$, and $L(i) > 0$. $L(i)$ strictly decreases in i if $i < i_0$; and may decrease or increase in i if $i > i_0$. Suppose $\mu(0,i) < i$, then $\mu(i) = \mu(0,i)$, and $L(i) = 0$. $\mu(i)$ strictly decreases in i if $i < i_0$; and may increase or decrease in i if $i > i_0$.*

3.2.2 Changes in matching efficiency

In this section, we examine how matching efficiency m affects the operation of the middleman. We start by showing how the curves $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ change as m decreases, i.e., matching efficiency improves. Figure 4 illustrates that as m decreases from m_1 to m_2, the incremental profits $\Delta\pi(\lambda, c)$ decrease, causing the curve $\Delta\pi(\lambda, c) = 0$ to shift downwards. At the same time, since the middleman is more likely to match with consumers early, the liquidity contribution of suppliers improves. As a result, $\theta_F(\lambda, c) = 0$ curve rotates upwards.

$\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ intersect at $(c_0, \lambda_0) = (k + u - \sqrt{k^2 + 4uk}, \frac{k+\sqrt{k^2+4ku}}{2mu})$. Note that c_0 does not depend on m. This implies that, as m decreases, the two curves intersect along the horizontal line of $c = c_0$, and the intersection point moves to the right. The intersection point lies within the set Ω as long as $\lambda_0 \leq 1$, or equivalently,

$$m \geq \tilde{m} \equiv \frac{k + \sqrt{k^2 + 4uk}}{2u}. \tag{19}$$

When $m > \tilde{m}$ (and $\bar{c}(i) > c_0$), all the three regions of (13) are not empty, as illustrated in Figure 1, that is, there are suppliers in $\Omega(i)$ with positive $\Delta\pi(\cdot)$ and negative $\theta_F(\cdot)$. Thus, liquidity cross-subsidization may arise.

In contrast, when $m \leq \tilde{m}$, the $\Delta\pi(\cdot) = 0$ curve lies entirely below the $\theta_F(\cdot) = 0$ curve. Figure 5 illustrate the case of $m = \tilde{m}$ where $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ intersect at $\lambda = 1$. In this case, all suppliers who bring a positive $\Delta\pi$ (the shaded region) also give the middleman a positive liquidity contribution, leading to $\mu = 0$. Indeed, a necessary condition for a binding liquidity constraint for the middleman is $m > \tilde{m}$.

[9] Because $\mu(0,i) < i$, $c = b(\lambda, \mu(0,i))$ must lie above $c = \bar{c}(i)$ if $\mu(0,i) > i_0$. But by our definition of $\mu(L,i)$ (see proof of Lemma 1), $\mu(0,i)$ in this case must be smaller than i_0.

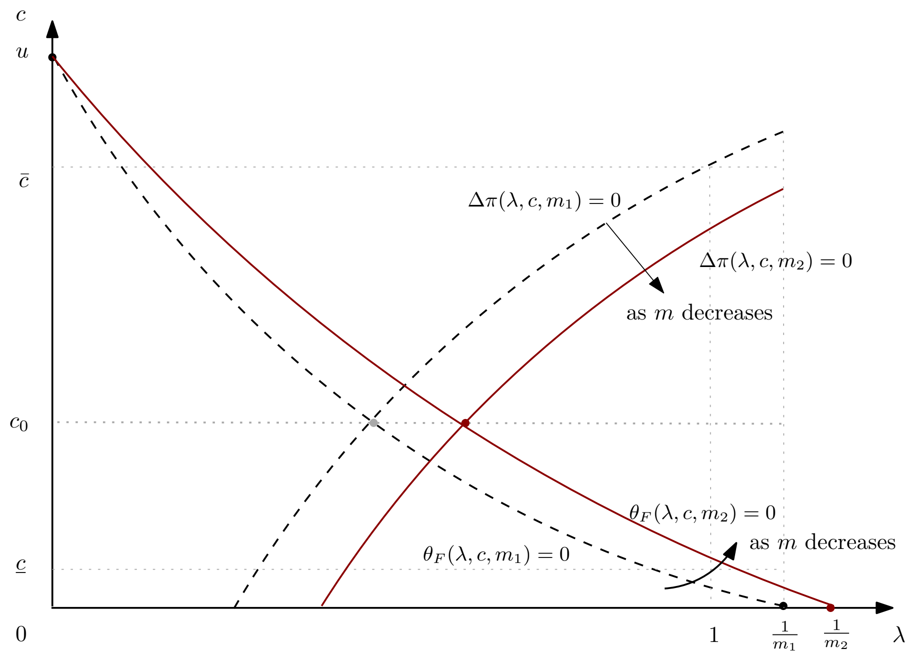


Figure 4: Effects of a decrease in m on $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$

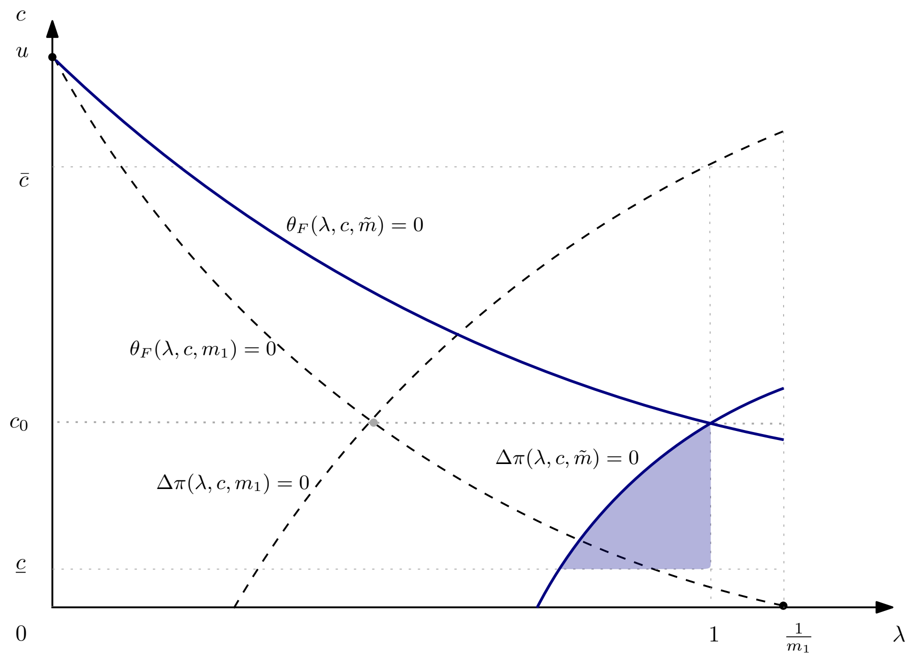


Figure 5: The curves of $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$ when $m = \tilde{m}$ and $m = m_1 > \tilde{m}$

Matching efficiency and the middleman's financing decision. We see in Lemma 2 that middleman finance is active if and only if $m > \frac{k}{(u-\underline{c})/2}$. If $m \in \left(\frac{k}{(u-\underline{c})/2}, \tilde{m}\right)$, all suppliers selected for finance contracts contribute positive liquidity, and no liquidity cross-subsidization occurs between suppliers. Since the liquidity constraint is not binding, i.e., $\mu(0,i) = 0$, the selection rule for financing suppliers is simply based on $\Delta\pi(\lambda, c) \geq 0$. Therefore, as m decreases, the set of financed suppliers becomes smaller.

Next, we consider $\mu(0,i) > 0$, which necessarily implies $m > \tilde{m}$. We are particularly interested in scenarios where the middleman's liquidity holding and supplier selection are both interior solutions. An interior liquidity holding ($L > 0$) indicates $\mu(0,i) > i$, and thus $\mu(i) = i$. An interior supplier selection requires $i < i_0$ (note that if $i \geq i_0$, then all participating suppliers are financed). These scenarios are depicted in Figures 2(a). We postpone the discussion of other cases (Figure 2(b) and Figure 3) to the end of the section.

In Figure 2(a), when m decreases, both the $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ curves shift to the right. Consequently, the selection curve $c = b(\lambda, \mu)$ also shifts to the right. This indicates that fewer suppliers are financed as the middleman's efficiency increases.

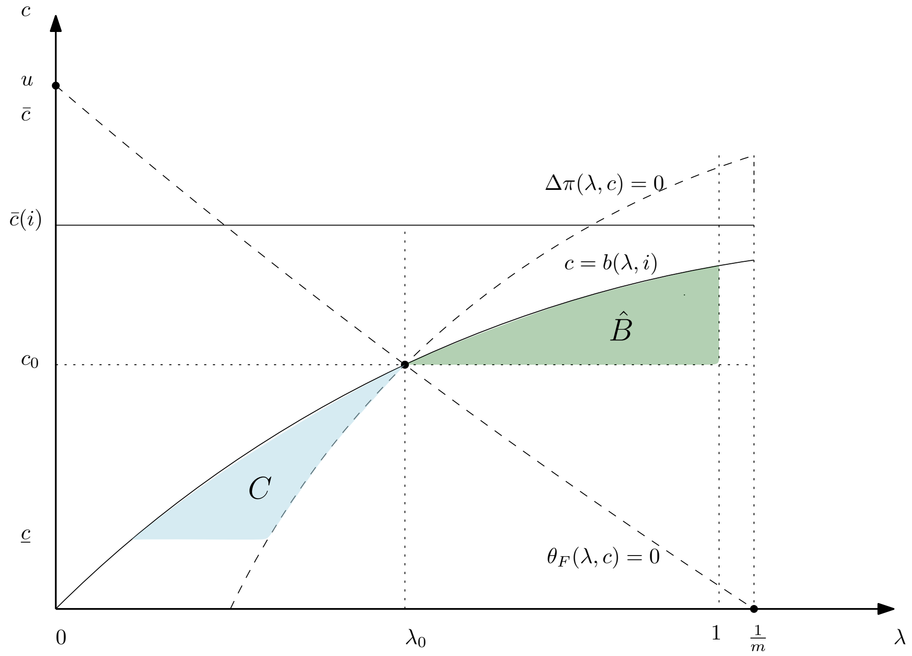


Figure 6: The two sets of suppliers that are affected by a change in m

The economics of this result follow from comparing the returns (costs) of financing a supplier $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)}$ with the liquidity value $\mu(i) = i$. Consider first the set of financed suppliers with $\Delta\pi > 0$,

$\theta_F < 0$ and $c > c_0$, denoted by region $\hat{B}$ in Figure 6. It is straightforward to derive that

$$\frac{\partial \frac{\Delta\pi(\lambda,c)}{-\theta_F(\lambda,c)}}{\partial m} = \frac{1}{-\theta_F}\frac{\lambda(u+c)}{2}\left(\underbrace{\frac{u-c}{u+c}}_{\hat{\imath}(c)} - \underbrace{\frac{\Delta\pi(\lambda,c)}{-\theta_F(\lambda,c)}}_{\hat{\mu}(\lambda,c)}\right) \tag{20}$$

where we have defined $\hat{\imath}(c)$ as the highest nominal interest rate at which consumers are still willing to purchase the good that is produced at cost c, and $\hat{\mu}(\lambda,c)$ as the liquidity value of the supplier indexed by (λ,c). For suppliers in $\hat{B}$, $\hat{\imath}(c) > \hat{\mu}(\lambda,c)$ (see the proof of Proposition 2). Given that $\theta_F(\lambda,c) < 0$, we have $\frac{\partial\left(\frac{\Delta\pi(\lambda,c)}{-\theta_F(\lambda,c)}\right)}{\partial m} > 0$. As a result, suppliers in set B_1 provide a lower return of $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)}$ to the middleman as m decreases. Consequently, marginal suppliers on the selection curve $c = b(\lambda,i)$ with $c > c_0$ (belonging to set $\hat{B}$) generate returns of $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)} < i$, making them no longer profitable for the middleman to finance.

For suppliers in set C of Figure 6, $\hat{\imath}(c) > i_0 > \hat{\mu}(\lambda,c)$.[10] By using (20) and $\theta_F(\cdot) > 0$, we have $\frac{\partial\left(\frac{\Delta\pi(\lambda,c)}{-\theta_F(\lambda,c)}\right)}{\partial m} < 0$. As a result, suppliers on the curve $c = b(\lambda,i)$ with $c < c_0$ experience an increase in funding costs, leading to $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)} > i$. These suppliers become too expensive as the liquidity source for the middleman. Since, in either case, the marginal suppliers are no longer financed, the set of financed suppliers shrinks.

Proposition 2 *Suppose $\mu(i) = i$ and $i < i_0$ (i.e., the middleman's liquidity and finance decisions are described by an interior solution). Then, a decrease in m (i.e., an increase in middleman's matching efficiency) leads to a smaller set of financed suppliers.*

We shall briefly mention other cases (not covered by the above proposition) where the middleman's liquidity holding or(and) the selection of suppliers takes corner solutions. In these cases, the set of financed suppliers does not necessarily contract as m decreases. Figure 2(b) illustrates a case where all suppliers participating in the DM are financed. This happens because $\mu(0,i) \geq i > i_0$ and $c = b(\lambda,\mu)$ consistently lies above $\bar{c}(i)$. In this scenario, changes in m have no effect on the scope of finance contracts.

In Figure 3, the middleman's liquidity holding gets corner solution of $L = 0$, implying that $\mu(i) = \mu(0,i)$ rather than $\mu(i) = i$. In the case shown in Figure 3(a), where $i < i_0$, the liquidity value $\mu(0,i)$ may either increase or decrease as m decreases. Thus, even though marginal suppliers generate lower returns and become more costly for the middleman, it can still be profitable to include them in finance contracts. As a result, the set of selected suppliers may either expand or contract. By contrast, in Figure 3(b), where $\bar{c}(i) < c_0$, $\mu(i) = \mu(0,i)$ must decrease as m decreases.[11] As a result, the set of financed suppliers contracts as m decreases.

[10] $\hat{\imath}(c) > i_0$ follows from that these suppliers locate below c_0. $i_0 > \hat{\mu}(\lambda,c)$ follows from the fact that the selection curve is upward-sloping.

[11] This can be proven by contradiction: if $\mu(0,i)$ increases or remains constant, the total liquidity Θ becomes positive after a small decrease in m, since the $\theta_F(\lambda,c)$ of every selected supplier increases, leading to a contradiction.

3.3 Welfare

In this section, we first examine the social optimum in the economy and demonstrate that a profit-maximizing middleman cannot achieve this optimum. However, the middleman is welfare improving. Further, somehow surprisingly, we find that social welfare can be improved with a higher nominal interest rate in the presence of the middleman. We also provide a sufficient condition under which social welfare is non-monotonic in nominal interest rate.

Social optimum. Consider a planner who chooses suppliers in the set Ω and assigns them to direct selling, a middleman contract, or a middleman-finance contract. Additionally, the planner also chooses a nominal interest rate. In this planner's problem, setting nominal interest rate $i = 0$ is dominant as it eliminates the liquidity constraint. Given $m < 1$, the middleman is more efficient than individual suppliers in matching demand with goods. Therefore, the planner includes all suppliers either in a middleman contract or a middleman-finance contract. In the following, we examine the planner's decision regarding whether or not to finance a supplier. We will examine under what conditions the planner chooses to finance a supplier and compare it to the choice of a profit-maximizing middleman.

Let $I(\lambda, c)$ be a binary function which equals one if a supplier of (λ, c) is financed. The total welfare per period is given by

$$\mathcal{W} = \int_{\Omega} \left\{ I(\lambda, c)(u - c - k) + \Big(1 - I(\lambda, c)\Big)(1 - m\lambda)(u - c) \right\} dG.$$

The total surplus for the goods is $u - c - k$ if the supplier is financed and is $(1 - m\lambda)(u - c)$ if not. The constrained efficient allocation is

$$I(\lambda, c) = \begin{cases} 1 & \text{if } \Delta v(\lambda, c) \equiv m\lambda(u - c) - k \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where we have defined $\Delta v(\lambda, c)$ as the surplus gain when a supplier (λ, c) is funded (note that $\Delta\pi(\lambda, c)$ is part of the surplus that goes to the middleman.) The middleman does not achieve the maximum welfare because the middleman ignores the positive externality to consumers when a supplier is financed. For example, at $i = 0$, the profit-maximizing middleman only finances a supplier if $m\lambda(u - c)/2 - k \geq 0$, disregarding the potential total surplus of $m\lambda(u - c) - k$ that could be achieved.

For any $i < i_2$, observe that the welfare change when the middleman provides active finance services is given by

$$\begin{aligned} \Delta\mathcal{W}(i) &\equiv \int_{\Omega(i)} \left\{ q(\lambda, c)(u - c - k) + \Big(1 - q(\lambda, c)\Big)(1 - m\lambda)(u - c) \right\} dG - \int_{\Omega(i)} (1 - m\lambda)(u - c) dG \\ &= \int_{\Omega(i)} q(\lambda, c)\Delta v(\lambda, c) dG. \end{aligned}$$

Since $\Delta\mathcal{W}(i) > \int_{\Omega(i)} q(\lambda,c)\Delta\pi(\lambda,c)dG$, whenever the latter is positive, funding suppliers is always welfare improving.

Nominal interest rates and welfare. We show that deviating from the Friedman rule may increase social welfare through liquidity cross-subsidization. We shall focus on the case that $\mu(0,i) > 0$ for $i \in [0,\varepsilon)$ where ε is small positive number. Otherwise, the outcome is trivial, i.e., independent of the nominal interest rate, the middleman always and only finances those with $\Delta\pi > 0$. We consider a marginal increase in i from $i = 0$, which together with $\mu(0,i) > 0$ implies $\mu(i) = i$ (see (17)). This situation is depicted in Figure 7. The grey region represents the set of suppliers, namely Ω. Relevant sets of suppliers are denoted by capital letters.

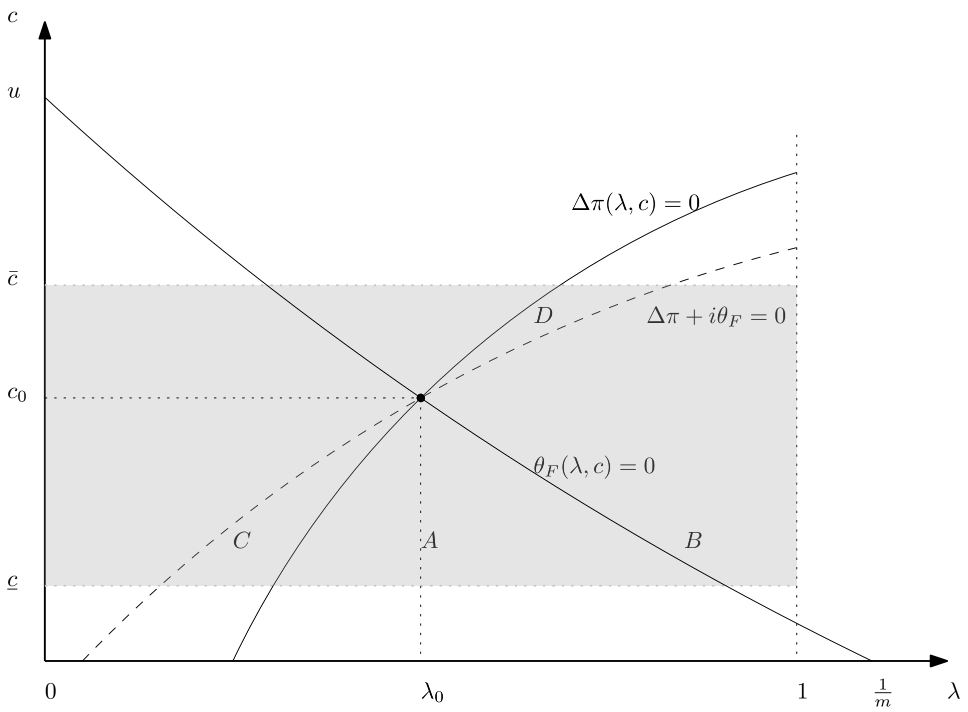


Figure 7: Friedman rule and welfare

When the nominal interest rate is zero, the middleman finances all suppliers with positive profitability, covering regions A, B, and D in the figure. As the interest rate rises, the middleman faces higher funding costs, then liquidity cross-subsidization among suppliers becomes profitable. This process involves excluding suppliers with positive profitability but negative liquidity (region D), while including suppliers with negative profitability but positive liquidity (region C). Consequently, the middleman's profits decrease. Suppliers' profits do not change since they are indifferent between being funded and not funded by the middleman. However, there is a potential for an increase in consumer surplus if the total trading volume increases. Ultimately, whether social welfare is improved or not depends on the dominance of the consumer surplus effect.

To analyze the impact on trading volume, let $\lambda^{\pi}(c)$ represent the combinations of (λ, c) for which $\Delta\pi(\lambda, c) = 0$, and let $\lambda^{\mu}(c)$ represent the combinations of (λ, c) such that $\Delta\pi(\lambda, c) + \mu\theta(\lambda, c) = 0$. Excluding suppliers in region D leads to a decreased trading volume given by:

$$m \int_{c_0}^{\bar{c}} \int_{\lambda^{\pi}(c)}^{\lambda^{\mu}(c)} \lambda g(\lambda, c) d\lambda dc,$$

while including suppliers in region C leads to an increased trading volume given by:

$$m \int_{\underline{c}}^{c_0} \int_{\lambda^{\mu}(c)}^{\lambda^{\pi}(c)} \lambda g(\lambda, c) d\lambda dc.$$

This is because, for instance, each of the newly added suppliers, measured by $\int_{c_0}^{\bar{c}} \int_{\lambda^{\pi}(c)}^{\lambda^{\mu}(c)} g(\lambda, c) d\lambda dc$, will become available to consumers even when he is hit by a liquidity shock, which occurs with probability $m\lambda$.

Comparing the above two volumes, we can see that when c_0 is large enough, the former volume can be made arbitrarily small, while when c_0 is small enough, the latter volume can be made arbitrarily small. Thus, for a sufficiently large c_0, the consumer surplus effect dominates, and deviating from the Friedman rule improves welfare. In other words, c_0 crucial determines the number of suppliers to exclude (those who contribute to negative liquidity) and the number of suppliers to include (those who contribute to positive liquidity).

c_0 is determined as the intersection of $\theta_F(\lambda, c) = 0$ with $\Delta\pi(\lambda, c) = 0$. As k decreases, c_0 increases accordingly: for $k \to 0$, $c_0 \to u > \bar{c}$, and for $k \to \bar{k}$, $c_0 \to \underline{c}$. Therefore, with k sufficiently small, increased trading volume outweighs decreased trading volume.

A similar analysis applies to λ. If λ_0 is sufficiently large, according to the cross-subsidization strategy, as i marginally increases from $i = 0$, the middleman excludes fewer suppliers and includes more suppliers in middleman-finance contracts. Just like before, m determines λ_0. With m sufficiently small e.g., close to $\tilde{m}$, λ_0 is larger and closer to $\lambda = 1$. As a result, the increased trading volume outweighs the decreased trading volume.

The following proposition provides sufficient conditions for critical values of k and m such that the decreased trading volume is smaller than the increased trading volume if k or m is lower than the critical value. Social welfare increases because there is a significant improvement in consumer surplus that outweighs the decrease in the middleman's profits. Hence, deviating from the Friedman rule is welfare-improving.

Proposition 3 *Let $\kappa \equiv \frac{k}{u} \in (0, \frac{\bar{k}}{u})$ and define $\tilde{m}(\kappa) = \frac{1}{2}\left(\kappa + \sqrt{\kappa^2 + 4\kappa}\right)$. Suppose that (λ, c) follows a uniform distribution and $\mu(0,0) > 0$. There exists a critical value $\kappa^* \in (0, \frac{\bar{k}}{u}]$ and $m^*(\kappa) \in (\tilde{m}(\kappa), 1]$, such that under $m < m^*(\kappa)$ or $\kappa < \kappa^*$ that marginally increases i from $i = 0$ improves welfare.*

Since as i continues to increase beyond i_2, the social welfare ultimately decreases to zero. The proposition also establishes sufficient conditions for a non-monotonic impact of i on welfare. Fig-

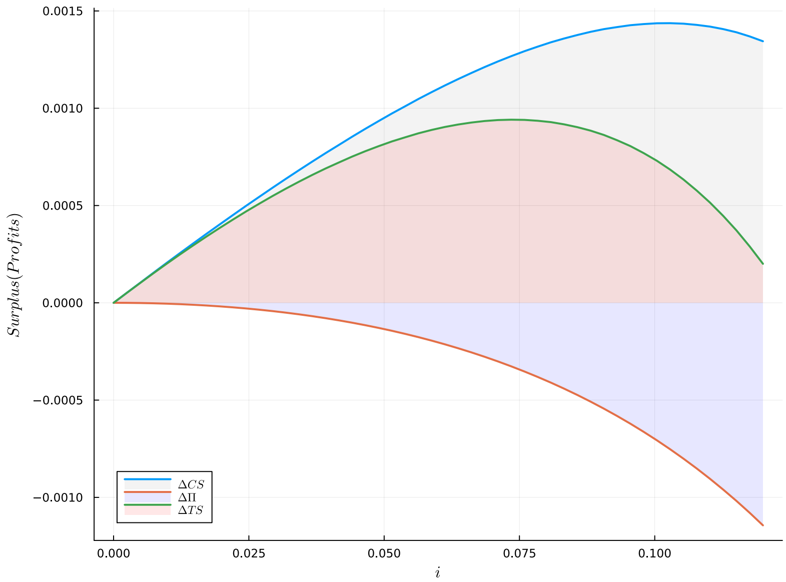


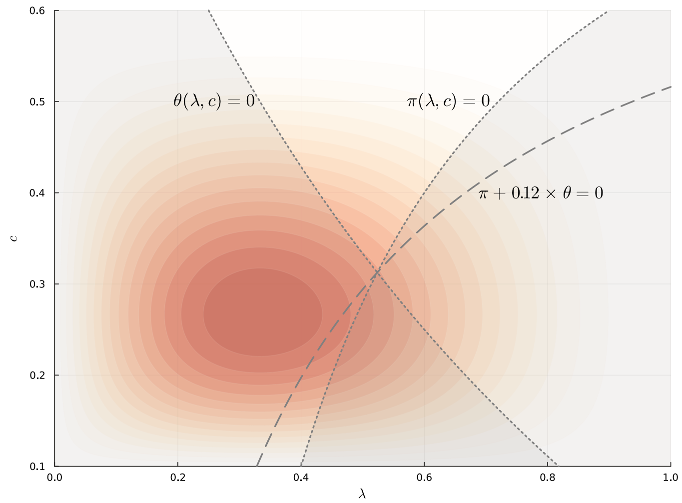
Figure 8: Welfare is non-monotonic in i under uniform distribution of (λ, c)

ure 8 illustrate how welfare changes with i using a numerical example with $u = 1, k = 0.1, m = 1, \underline{c} = 0.1, \bar{c} = 0.6$, and a uniform distribution of (λ, c). Under these values, $\mu(0, i) = 0.26$ for $i \leq i_1 = 0.25$. The figure shows that (1) the aggregate profits ($\Delta\Pi$, red curve) exhibit a monotonic decrease in i due to the exclusion of suppliers with positive π and the inclusion of suppliers with negative π; and (2) the total consumer surplus (ΔCS blue curve) follows an inverted U-shape because total trading volume first increases and then decreases. The effect of consumer surplus dominates. Consequently, the total surplus (ΔTS green curve) first increases and then decreases at relatively higher levels of i.

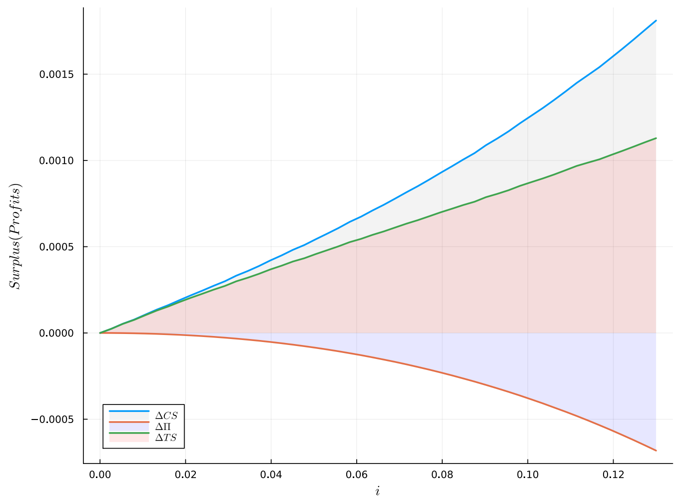
Of course, the suboptimality of the Friedman rule can occur with non-uniform distributions. Figure 9 provides a numerical exercise with $u = 1, k = 0.18, m = 1, \underline{c} = 0.1, \bar{c} = 0.6$, and both λ, and c follow a $Beta(2, 3)$ distribution. Under these values, $\mu(0, i) = 0.137$ for $i < i_1 = 0.25$. Panel (a) shows the implied densities (contour graph in red) and a particular selection rule of $\pi + 0.12 \times \theta = 0$. Panel (b) shows that, as i increases, the blue curve representing total consumer surplus (ΔCS) increases monotonically, which outweighs the decrease in total profits ($\Delta\Pi$, red curve), resulting in a monotonically increasing total surplus (ΔTS, green curve) for the shown range of nominal rate.

4 Suppliers' access to money market

In this section, we present an extension of the model where suppliers have access to the money market and can hold money to prepare for liquidity needs. Suppose now that suppliers are infinitely lived and have a discount factor $\beta^s \in (0, \beta]$. The nominal interest rate is $i = \frac{\gamma}{\beta} - 1$,



(a) Density and selection rule ($\mu = 0.12$)



(b) Welfare change

Figure 9: Welfare increases in i if $(\lambda, c) \sim Beta(2, 3)$

while the effective nominal interest rate faced by suppliers is $i^s = \frac{\gamma}{\beta^s} - 1$. Let $\Delta i^s = i^s - i \geq 0$ be the premium. In the following analysis, we focus on the parameter space where $\bar{c} > c_0 > \underline{c}$ and $\lambda_0 < 1$ (see footnote 6). To isolate our point from the influence of $\bar{c}(i)$, we will consider $i \leq i_1$.

Let $z^s = z^s(c)$ represent the real balance held by a supplier with cost c. A supplier (λ, c) with $z^s(c)$ real balance has a DM value given by

$$W^s\left(z^s + \left((1-\lambda) + \lambda \min\{\frac{z^s}{c}, 1\}\right)(p - c)\right) = z^s + \left((1-\lambda) + \lambda \min\{\frac{z^s}{c}, 1\}\right)\frac{u-c}{2} + W^s(0).$$

The term $\lambda \min\{\frac{z^s}{c}, 1\}$ indicates that if the supplier is hit by a liquidity shock, he can still use his money holdings to produce and sell to $\min\{z^s/c, 1\}$ consumers. Since the supplier's DM value remains the same whether he sells his goods directly or through the middleman, the supplier's money-holding problem is

$$\max_{z^s}\Big\{-\gamma z^s+\beta^s V^s(z^s)\Big\}.$$

where $V^s(z^s)=W^s(\tilde{z}^s)$ and $\tilde{z}^s\equiv z^s+\Big((1-\lambda)+\lambda\min\{\frac{z^s}{c},1\}\Big)\frac{u-c}{2}$. Inserting $W^s(\tilde{z}^s)$, the problem can be rewritten as

$$\max_{z^s}\Big\{-\gamma z^s+\beta^s\Big(z^s+\Big[(1-\lambda)+\lambda\min\{\frac{z^s}{c},1\}\Big]\frac{u-c}{2}\Big)\Big\}$$

Obviously, suppliers never hold $z^s>c$. The first-order condition indicates that all the suppliers with (λ,c) satisfying

$$\beta^s\Big[\frac{\lambda(u-c)}{2}+c\Big]>\gamma c$$

should hold $z^s(c)=c$ money. Using $i^s=\gamma/\beta^s-1$, this condition can be written as

$$c<c^s(\lambda,i^s)\equiv\frac{\lambda}{\lambda+2i^s}u. \tag{21}$$

Therefore, suppliers with $c<c^s(\lambda,i^s)$ hold $z^s(c)=c$ and those with $c\geq c^s(\lambda,i^s)$ hold $z^s(c)=0$.

We now consider the middleman's problem. She can only invite suppliers who choose not to hold money to the finance contract. Given that $i<i_1$, we can ignore how $\bar{c}(i)$ changes. Therefore, the feasible set of suppliers depends only on i^s:

$$\tilde{\Omega}(i^s)=\{(\lambda,c)\in\Omega|c\geq c^s(\lambda,i^s)\}$$

which is nonempty. The middleman's supplier selection problem for the finance service is given by:

$$\max_{\{q(\cdot)\}_{(\lambda,c)\in\tilde{\Omega}(i^s)}}\int_{\tilde{\Omega}(i^s)}q(\lambda,c)\Delta\pi(\lambda,c)dG,$$

subject to the liquidity constraint:

$$\int_{\tilde{\Omega}(i^s)}q(\lambda,c)\theta_F(\lambda,c)dG+L\geq 0,$$

where i^s and L are taken as given.

In the previous sections, we demonstrated that finance contracts are always profitable for some suppliers in a money equilibrium when $i\leq i_1$ (which is less than i_2) and $\lambda_0<1$, because region A in Figure 1 is non-empty (see Lemma 2 for a weaker condition). However, under the same conditions, when suppliers have access to market liquidity, middleman-finance contracts may not always be activated.

Proposition 4 *Suppose $\lambda_0<1$, $\underline{c}>0$, $i<\min\{i_1,\frac{k\bar{\lambda}}{mu\bar{\lambda}-2k}\}$, and suppliers can access the money market at an effective interest of $i^s\geq i$. Then there exists $i<\underline{i}^s<\bar{i}^s\equiv\frac{(u-c)\bar{\lambda}}{2\underline{c}}$ such that:*

- *If $i^s \leq \underline{i}^s$, suppliers with $c \leq c^s(\lambda, i^s)$ hold money to meet their liquidity needs, and middleman finance remains inactive.*
- *If $i^s \geq \bar{i}^s$, no supplier holds money, and middleman finance becomes active for some suppliers.*
- *If $i^s \in (\underline{i}^s, \bar{i}^s)$, suppliers with $c \leq c^s(\lambda, i^s)$ holds money while middleman finance is active for some other suppliers.*

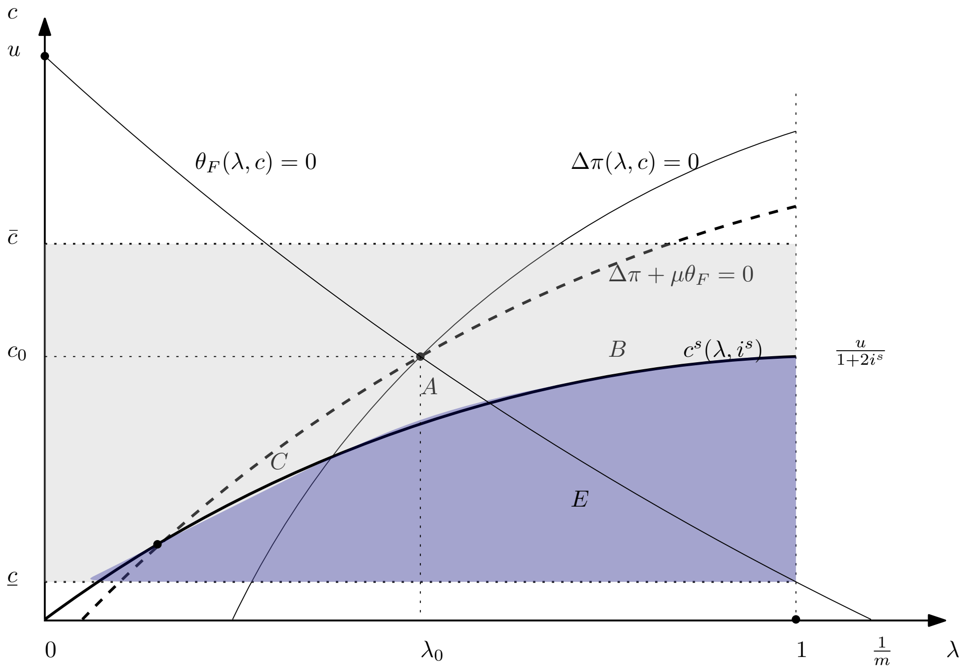


Figure 10: Suppliers' money holdings coexist with the middleman-provided finance

Figure 10 illustrates the third scenario, where suppliers with costs below $c^s(\lambda, i^s)$ opt to hold money individually (represented by region E in dark blue), thereby refraining from entering into the middleman's finance contract. In contrast, suppliers with costs above $c^s(\cdot)$ choose not to hold money. Suppliers within regions A, B, and C, however, participate in the finance contract.

Of particular interest is the scenario where $i^s = i$, namely, suppliers face the same nominal interest rate as the middleman. The main concern is whether suppliers are still financed by the middleman in equilibrium. To address this issue, we need to extend the range of i to $[0, i_2]$. For simplicity of exposition, we consider the limit as $m \to 1$, i.e., middleman and suppliers have the same matching capacity. Our conclusion holds for $m < 1$.

We have two results. First, in a monetary equilibrium, there always exists a set of suppliers who choose to hold money. These suppliers are still characterized by $c < c^s(\lambda, i)$. Note that for all $i < i_2$, it holds that $\underline{c} < c^s(1, i)$, indicating that such a set of suppliers is non-empty. Second, active middleman-offered finance coexists with suppliers' individual money holdings when the middleman has a small k, *and* the nominal interest rate takes some intermediate values. Small k's are needed because otherwise, all suppliers in the feasible set give negative profits. Not too low i's are needed because otherwise, all the suppliers with a positive profit $\Delta\pi$ find it less costly to hold money by themselves than using middleman finance. Not too high i's are needed because otherwise, consumers opt only for those suppliers with $c < \bar{c}(i)$, and among these suppliers, those with a positive profit $\Delta\pi$ will choose to hold money by themselves.

Proposition 5 *Suppose $m \simeq 1$ and $k < u/6$. For intermediate values of $i = i^s < i_2$, middleman finance coexists with suppliers' money holdings.*

5 Discussions

In this section, we explore three distinct examples of middlemen providing liquidity: supplier finance, keiretsu, and rural credit cooperatives.

5.1 Supplier Finance Programs

Our model speaks directly to supplier finance, a contemporary example of middlemen who act as liquidity providers. For instance, in 2020, Coop, a leading supermarket chain in UK, embarked on an innovative supplier finance program in partnership with the fintech platform PrimeRevenue. This initiative was designed to ease cash flow constraints of a diverse range of suppliers, from small-scale farms to relatively large manufacturing companies, which became serious especially during the pandemic.

Under this arrangement, Coop invites eligible suppliers from its existing network and extends the payment terms for the selected suppliers. In return for the delayed payments, these suppliers are granted access to an online portal where they can request early payments from PrimeRevenue. At the end of the extended payment term, Coop repays PrimeRevenue for the funds advanced to the suppliers. This approach allows suppliers to receive timely and affordable financing while Coop optimizes its cash flow and deepens its relationships with its suppliers. Our model well captures the operation of this supplier finance.

The widespread success of supplier finance programs, such as Coop's, has been largely driven by advancements in financial technology (fintech). While the concept of supplier finance is not new—it originated in the early 1980s with Fiat, an Italian automobile manufacturer, in collaboration with the Italian bank Mediocredito Centrale—it was initially a manual and labor-intensive process, involving significant paperwork that limited the scalability of these programs. Fintech innovations have transformed supplier finance by enabling real-time data sharing and seamless integration with enterprise resource planning (ERP) systems. For Coop, the adoption of supplier finance was made possible by the advanced, cost-effective technology provided by PrimeRevenue. This Fintech service provider allows suppliers to easily upload invoices, receive early payments at competitive rates, and manage their cash flow more effectively. In our model, this advancement in fintech is captured by a lower lump-sum cost, k, associated with processing each supplier.

The same reasoning (lower k) explains why, in emerging market economies, e-commerce platforms have become the most successful retailing middlemen in the supplier finance business. Notable examples include Alibaba and JD.com in China, MercadoLibre in Latin America, and Flipkart in India. These platforms utilize their technological advantages to offer cost-effective financial solutions, positioning them as ideal candidates for implementing supplier finance in markets with diverse levels of financial sophistication.[12]

Next, we discuss how our analysis captures the key features of supplier finance in more detail. We focus on three critical ones: the selection of suppliers, the cross-subsidization of liquidity among different market participants, and the interplay between the middleman's efficiency in retail markets for matching supply and demand and the scope of financed suppliers.

1. Selecting suppliers. Highly selected participants are a common feature of real-world supplier finance programs. In the case of Coop, fewer than 100 suppliers were selected when the supplier finance program was launched in 2020, although Coop had almost 2400 stores in the UK and thousands of suppliers. Similarly, Amazon Lending, a supplier finance program offered to third-party merchants on the Amazon platform, follows an invitation-only approach, providing customized credit amounts and terms tailored to the specific needs and situations of each seller.[13]

Among various factors to be considered when selecting suppliers, we have specifically modeled each supplier's profit and liquidity contribution. This echoes the common practice of supplier segmentation and prioritization in supply chain optimization. The *Supply Chain Finance Knowledge Guide* published by the International Finance Corporation states that to implement supplier finance, suppliers should be prioritized based on their *relationships with the buyer firm* (i.e., the middleman in our model) and *financial needs*. Suppliers with strong, stable, and long-term relationships with the buyer firm tend to be crucial to the buyer firm's value creation, corresponding to a large and positive π in the model. The likelihood of financial needs is captured by λ. For instance, in the Amazon Lending program mentioned above, merchants with a proven track record of growing sales and high customer satisfaction are more likely to be invited.

[12]Our model captures the essential features of various financial arrangements within the broader definition of supply chain finance, including pre-shipment finance, distributor finance, and dynamic discounting. In pre-shipment finance, suppliers have the option to receive an upfront payment for verified purchase orders, enabling them to access liquidity before the goods are shipped. In distributor finance, distributors of large corporations receive funding to cover inventory holding costs and bridge the liquidity gap until they receive sales revenue. In dynamic discounting, the buyer and supplier negotiate a discount rate based on payment timing. If the supplier accepts the early payment offer, the receivable is reduced. In all these arrangements, the middleman works with a diverse group of suppliers/distributors and adjusts payment terms strategically to pool liquidity. The middleman then takes advantage of the liquidity pool to fund suppliers/distributors requiring immediate funding.

[13]For more details of Coop's supplier finance program, see *https://scfcommunity.org/briefing/news/2020-retail-and-apparel-winner-co-operative-group/*. For model details about Amazon Lending, see *https://www.junglescout.com/blog/amazon-lending-program*. All links were accessed on Jul 17, 2023.

2. Liquidity cross-subsidization. JD.com provides a compelling real-world example of liquidity cross-subsidization. JD.com, the leading e-commerce platform in China, launched a supplier finance program known as ”JingBaoBei” in 2014. JingBaoBei targets all JD's suppliers, including those in the direct selling channel and third-party merchants on the platform. JingBaoBei allows these suppliers to request advance payment based on their accounts receivable from JD.com. From 2014 to 2023, JingBaoBei provided funding to over 200,000 vendors with a total amount of more than 730 billion RMB.

JingBaoBei is mainly funded by pooled liquidity from suppliers. Prior to 2016, JingBaoBei relied solely on JD's self-funding and, in particular, on suppliers' trade credit, which can be traced by JD's financial reports. For instance, in 2021, the increase in accounts payable alone constituted more than 77% of the net cash inflow of JD's operating activities. JD's cash conversion cycle also confirms that JD sources significant cash inflows through the use of suppliers' trade credit. A simple calculation reveals that JD can freely use suppliers' trade credit for more than 20 days.[14]

In 2016, JD introduced other funding for JingBaoBei through asset-backed securities, akin to the liquidity holdings L in our model, with the underlying assets being suppliers' accounts receivable. Despite this, JD's self-funding continues to be the main funding source for JingBaoBei.

It is worth mentioning that there exists a group of suppliers that offer trade credit to JD.com, but almost never ask for funds from JingBaoBei. Indeed, while JingBaoBei can be an important source of liquidity for small and medium-sized suppliers that are constantly under the pressure of liquidity needs (corresponding to those with large λ's in the model), large manufacturing firms like Lenovo, Philips, and Bosch that supply directly to JD.com rarely use JingBaoBei if any. These suppliers correspond to those with small λ's in the model and subsidize liquidity to other suppliers in JD's supply chain.

3. Matching efficiency of middlemen, growing demand for liquidity, and expansion of supplier finance. Our model highlights that both the matching efficiency of middlemen and the liquidity needs of suppliers are key factors driving the growth of supplier finance. Indeed, this seems to be a trend that has become pronounced recently. For example, during the pandemic, many retailers (acting as middlemen) faced significant disruptions, such as lockdowns and various frictions in the retail market, which led to increased inventories. Naturally, many companies extended their payment terms to suppliers–in 2021, payables outstanding amounted to 62.2 days on average, according to a survey of the largest 1,000 U.S. companies. To ensure that suppliers maintained sufficient cash flow for the timely delivery of goods and services, retailers increas-

[14]From 2015 to 2018, JD's accounts payable turnover days have gone up from 41.9 to 58.1 days. This means, for instance, in 2018, it took more than on average 58 days for JD to pay off its suppliers. On the other hand, JD's accounts receivable turnover is quite short, with payments being received from customers within five days of a sale. Combining these numbers with a 30-day inventory turnover, JD can efficiently use supplier trade credit for about 23 ($= 58 - 5 - 30$) days before having to pay it off. Notably, this strategy has proven successful for JD, as its cash position has consistently improved alongside its total revenue.

ingly relied on supplier financing.[15]

In our model, exogenous disruptions in retail markets during the pandemic can be captured by an increase in the parameter m, which represents the middleman's reduced efficiency in matching with consumers. Proposition 2 shows that in the relevant region of parameters, as m increases, the scope of suppliers offered finance should expand. This result is in line with the observed behavior of retail markets.

The pandemic may not be the only driving force of a growing liquidity demand by suppliers. For instance, the inventory turnover days for sellers on Taobao, China's leading e-commerce platform owned by Alibaba, increased from 5.87 in 2017 to 20.33 in 2023. This suggests a reduced likelihood of suppliers matching with consumers on Taobao, again captured by an increase in m. Since this trend already started before the pandemic, it could be driven by other factors, e.g. the influx of new sellers on the platform and the emergence of competing platforms in China, such as JD.com and Pinduoduo. In line with our result, the supplier finance program by Taobao (specifically Ant Finance, Alibaba's financial subsidiary) expanded significantly, with credit balances rising from 647.5 billion RMB in 2017 to 2,153.6 billion RMB in 2020. This positive correlation between the inventory turnover days of Taobao and the scale of its supplier finance program is illustrated in Figure 11.[16]

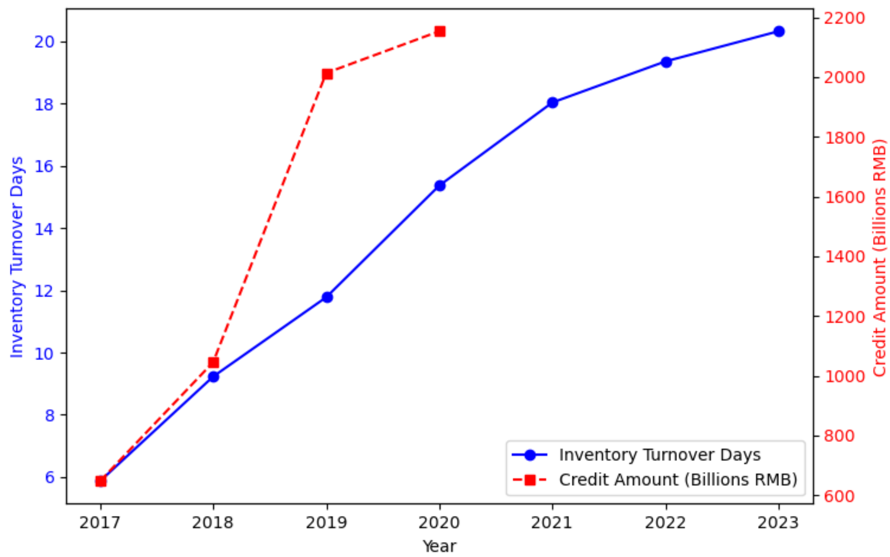


Figure 11: Inventory turnover (days) and credit amount on Alibaba platform (2017 - 2023)

[15]For example, Constellation Brands Inc., a New York-based producer of beer, wine, and spirits—including brands such as Corona beer and Svedka vodka—launched a supplier finance program in 2022 in response to significant inventory growth and extended days of payables outstanding. Similarly, VF Corp., the parent company of popular brands such as Vans, North Face, and Supreme, initiated a supplier finance program in 2022 under similar circumstances. For further information, see the Wall Street Journal report at *https://www.wsj.com/articles/companies-offer-supply-chain-financing-to-vendors-as-they-bulk-up-on-inventory-push-out-payment-terms-11658316600?*, accessed on Jul 17, 2023.

[16]The inventory turnover days are obtained from *https://www.gurufocus.com/*. The credit balances of the supplier finance program are obtained from the *Ant Group Co., Ltd. Initial Public Offering and Listing on the STAR Market Prospectus*, and this variable is only available up to 2020.

5.2 Keiretsu

Our model also applies to keiretsu, a prominent organizational structure in the Japanese economy known for its inherent liquidity-sharing mechanisms. A keiretsu is a network of companies with interlocking business relationships and shareholdings, designed to foster stability and mutual support among its members. In a *horizontal keiretsu*, a commercial bank serves as the core entity, providing financial services and coordinating liquidity among the member companies. The "Big Six" horizontal keiretsu in Japan—Fuyo, Sanwa, Sumitomo, Mitsubishi, Mitsui, and DKB Group—are prime examples of this structure. Conversely, a *vertical keiretsu* links suppliers, manufacturers, and distributors within a specific industry, often with less direct influence from banks. Notable examples of vertical keiretsu include Toyota, Toshiba, and Nissan, where the main manufacturer plays a pivotal role in coordinating supply chain activities and ensuring liquidity.

While our model effectively captures aspects of horizontal keiretsu, the idea of middleman-provided finance aligns more naturally with vertical keiretsu structures. In these arrangements, the core manufacturer acts as the "middleman," managing funds to provide liquidity to its network of suppliers and distributors. Compared to supplier finance above, keiretsu networks typically involve a smaller number of firms and are deeply rooted in Japanese business culture, which emphasizes long-term relationships, mutual trust, and stability. This cultural foundation makes keiretsu less reliant on the advancements in financial technology. Instead, keiretsu relies more on established business practices and inter-firm relationships to facilitate liquidity sharing within the group.

An interesting feature of keiretsu is the joint financing initiatives, where member companies create shared financing vehicles, such as joint venture funds, investment funds, or specialized financing entities. These vehicles pool contributions from participating companies to create a collective source of financing. Through these shared financing vehicles, loans, equity investments, or other financial instruments can be extended to member companies within the keiretsu. Thus, the concept of liquidity cross-subsidization remains relevant in the context of keiretsu.

5.3 German Rural Credit Cooperatives

Credit cooperatives and microcredit institutions play a pivotal role as financial intermediaries in developing economies. A standout historical example is the German rural credit cooperatives of the 19th century. These credit cooperatives accepted deposits from members and made loans to members. By 1914, there were 19,000 credit cooperatives, accounting for approximately 7% of all German banking liabilities. The cooperatives exhibit several characteristics that align with our model (Guinnane, 2001), despite that cooperatives do not necessarily operate as middlemen.[17]

Like suppliers in our model, potential members of the German credit cooperatives faced high borrowing costs. The nation in the nineteenth century had a liberated yet undercapitalized peasantry. Before the advent of credit cooperatives, smallholders relied heavily on costly credit from informal lenders, often facing annual interest rates exceeding 30%. The emergence of cooperatives provided a much-needed alternative, offering more affordable credit options.

While modern supplier finance leverages information advantage due to buyer firms' close ties with suppliers, the German rural cooperatives relied on intimate community knowledge among members. Cooperatives deliberately limited their operations to compact geographic regions, often just one or two villages, and excluded residents from outside their designated area. This gives the cooperatives an in-depth understanding of the members' habits, character, and abilities, allowing a highly selective membership process based on this information. Not all applicants were granted membership, and even among members, not all were approved for loans. Any member exhibiting behaviors, such as excessive drinking, deemed detrimental to the cooperative's ethos could face expulsion.

Our model underscores significant heterogeneity among middleman's suppliers, which also holds among German cooperative members. As documented by Guinnane (2001), a cooperative called Diestedde, which operated for two villages, Diestedde and Stunnighausen, had 282 members. These members had different land types and farm sizes. For example, 61 members are large farmers, while 115 are small farmers. The Diestedde cooperative tailored the provision of credit, including loan sizes and terms, to these specific member profiles. Like suppliers in our model, the liquidity needs of members vary. In another cooperative called Diestedde, 56% of members had not borrowed any funds even several years after joining. In stark contrast, many members were granted loans on the very first day of their membership.

The prevalence of liquidity cross-subsidization among cooperative members is evident. For instance, more than half of the Diestedde cooperative members did not borrow during their initial five years of membership. This implies that these members essentially contributed funds to meet the liquidity needs of other members. In a similar vein, another cooperative called Hatzfeld exhibited a lower but still significant proportion, with one-fifth of its members being non-borrowers and serving as pure fund contributors.

A deeper exploration of these fund contributors reveals a remarkable resemblance to our model. In our model, the middleman acquires liquidity from suppliers who operate at a negative profit, essentially subsidizing them through retail revenue. Guinnane (2001) suggested that those who primarily contributed funds to cooperatives often had businesses dependent on the prosperity of their local community, such as shopkeepers or local artisans. As a result, these members'

[17] Our model accommodates this scenario. If the intermediary has no matching advantage, i.e., $m \geq 1$, then it is optimal for the intermediary to focus solely on operating the finance program.

funding contributions are also “subsidized” by other community members who purchase goods or services from them.

6 Conclusion

We developed a simple model of middlemen providing liquidity support to suppliers. This model captures key features observed in real-world scenarios, such as selecting among heterogeneous suppliers, pooling liquidity from suppliers, providing early payments to those with urgent liquidity needs, and the interplay between the middleman's matching advantage and its role as a financier. Our findings highlight the significance of liquidity cross-subsidization for the effective functioning of middleman liquidity provision and its overall welfare effect. We show that the nominal interest rate affects the trade-off between liquidity and profitability for selecting suppliers for middleman finance. We demonstrate that deviating from the Friedman rule may lead to welfare gains. When suppliers also have access to the money market, we investigate the coexistence of middleman-provided liquidity support and suppliers' holdings of liquidity.

References

ANDERSON, S., AND BEDRE-DEFOLIE, Ö. "Online Trade Platforms: Hosting, Selling, or Both?" *International Journal of Industrial Organization*, Vol. 84 (2022), 102–861.

ANDOLFATTO, D., BERENTSEN, A., AND MARTIN, F. M. "Money, Banking, and Financial Markets." *The Review of Economic Studies*, Vol. 87, 10 (2019), 2049–2086.

BERENTSEN, A., CAMERA, G., AND WALLER, C. "Money, Credit and Banking." *Journal of Economic Theory*, Vol. 135 (2007), 171–195.

BETHUNE, Z., ROCHETEAU, G., WONG, T.-N., AND ZHANG, C. "Lending Relationships and Optimal Monetary Policy." *The Review of Economic Studies*, Vol. 89, 11 (2021), 1833–1872.

BETHUNE, Z., SULTANUM, B., AND TRACHTER, N. "An Information-based Theory of Financial Intermediation." *The Review of Economic Studies*, Vol. 89, 12 (2022), 2381–2444.

BIGLAISER, G. "Middlemen as Experts." *The RAND Journal of Economics*, Vol. 24 (1993), 212–223.

BOTTAZZI, L., GOPALAKRISHNA, G., AND TEBALDI, C. "Supply Chain Finance and Firm Captial Structure." CEPR Discussion Paper DP18468, 2023.

BURKART, M., AND ELLINGSEN, T. "In-Kind Finance: A Theory of Trade Credit." *American Economic Review*, Vol. 94 (2004), 569–590.

CAMERA, G. "Search, Dealers, and the Terms of Trade." *Review of Economic Dynamics*, Vol. 4 (2001), 680–694.

CUÑAT, V. "Trade Credit: Suppliers as Debt Collectors and Insurance Providers." *The Review of Financial Studies*, Vol. 20 (2007), 491–527.

DEVALKAR, S. K., AND KRISHNAN, H. "The Impact of Working Capital Financing Costs on the Efficiency of Trade Credit." *Production and Operations Management*, Vol. 28 (2019), 878–889.

DIAMOND, D. W., AND DYBVIG, P. H. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, Vol. 91 (1983), 401–419.

DONG, M. "Inflation and Variety." *International Economic Review*, Vol. 51 (2010), 401–420.

ETRO, F. "Device-funded vs Ad-funded Platforms." *International Journal of Industrial Organization*, Vol. 75 (2021a), 102–711.

——— "Product Selection in Online Marketplaces." *Journal of Economics & Management Strategy*, Vol. 30 (2021b), 614–637.

——— "Hybrid Marketplaces with Free Entry of Sellers." *Review of Industrial Organization*, Vol. 62 (2023), 119–148.

——— "E-commerce Platforms and Self-preferencing." *Journal of Economic Surveys*, Vol. 38 (2024), 1516–1543.

Garcia-Appendini, E., and Montoriol-Garriga, J. "Firms as Liquidity Providers: Evidence from the 2007–2008 Financial Crisis." *Journal of Financial Economics*, Vol. 109 (2013), 272–291.

Gautier, P., Hu, B., and Watanabe, M. "Marketmaking Middlemen." *The RAND Journal of Economics*, Vol. 54 (2023), 83–103.

Giannetti, M., Burkart, M., and Ellingsen, T. "What You Sell Is What You Lend? Explaining Trade Credit Contracts." *The Review of Financial Studies*, Vol. 24 (2011), 1261–1298.

Gu, C., Mattesini, F., Monnet, C., and Wright, R. "Banking: A New Monetarist Approach." *The Review of Economic Studies*, Vol. 80, 01 (2013), 636–662.

Gu, C., Mattesini, F., and Wright, R. "Money and Credit Redux." *Econometrica*, Vol. 84 (2016), 1–31.

Guinnane, T. W. "Cooperatives as Information Machines: German Rural Credit Cooperatives, 1883-1914." *The Journal of Economic History*, Vol. 61 (2001), 366–389.

Hagiu, A., Teh, T.-H., and Wright, J. "Should Platforms be Allowed to Sell on Their Own Marketplaces?" *The RAND Journal of Economics*, Vol. 53 (2022), 297–327.

Johri, A., and Leach, J. "Middlemen and the Allocation of Heterogeneous Goods." *International Economic Review*, Vol. 43 (2002), 347–361.

Kang, Z. Y., and Muir, E. V. "Contracting and Vertical Control by a Dominant Platform." Working paper, 2022.

Kouvelis, P., and Xu, F. "A Supply Chain Theory of Factoring and Reverse Factoring." *Management Science*, Vol. 67 (2021), 6071–6088.

Lagos, R., and Wright, R. "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy*, Vol. 113 (2005), 463–484.

Li, F., Murry, C., Tian, C., and Zhou, Y. "Inventory Management in Decentralized Markets." *International Economic Review*, Vol. 65 (2024), 431–470.

Li, Y. "Middlemen and Private Information." *Journal of Monetary Economics*, Vol. 42 (1998), 131–159.

——— "Money and Middlemen in an Economy with Private Information." *Economic Inquiry*, Vol. 37 (1999), 1–12.

Madsen, E., and Vellodi, N. "Insider Imitation." SSRN working paper 3832712, 2023.

NOCKE, V., AND THANASSOULIS, J. "Vertical Relations under Credit Constraints." *Journal of the European Economic Association*, Vol. 12 (2014), 337–367.

NOSAL, E., WONG, Y.-Y., AND WRIGHT, R. "More on Middlemen: Equilibrium Entry and Efficiency in Intermediated Markets." *Journal of Money, Credit and Banking*, Vol. 47 (2015), 7–37.

PADILLA, J., PERKINS, J., AND PICCOLO, S. "Self-Preferencing in Markets with Vertically Integrated Gatekeeper Platforms." *The Journal of Industrial Economics*, Vol. 70 (2022), 371–395.

PETERSEN, M. A., AND RAJAN, R. G. "Trade Credit: Theories and Evidence." *The Review of Financial Studies*, Vol. 10 (1997), 661–691.

RHODES, A., WATANABE, M., AND ZHOU, J. "Multiproduct Intermediaries." *Journal of Political Economy*, Vol. 129 (2021), 421–464.

ROCHETEAU, G., WRIGHT, R., AND ZHANG, C. "Corporate Finance and Monetary Policy." *American Economic Review*, Vol. 108 (2018), 1147–86.

RUBINSTEIN, A., AND WOLINSKY, A. "Middlemen." *Quarterly Journal of Economics*, Vol. 102 (1987), 581–593.

SHEVCHENKO, A. "Middlemen." *International Economic Review*, Vol. 45 (2004), 1–24.

SHOPOVA, R. "Private Labels in Marketplaces." *International Journal of Industrial Organization*, Vol. 89 (2023), 102–949.

SMITH, E. "Intermediated Search." *Economica*, Vol. 71 (2004), 619–636.

SPULBER, D. F. "Market Making by Price-setting Firms." *The Review of Economic Studies*, Vol. 63 (1996), 559–580.

TIROLE, J., AND BISCEGLIA, M. "Fair Gatekeeping in Digital Ecosystems." TSE Working Paper, 2023.

TUNCA, T. I., AND ZHU, W. "Buyer Intermediation in Supplier Finance." *Management Science*, Vol. 64 (2017), 5631–5650.

WATANABE, M. "A Model of Merchants." *Journal of Economic Theory*, Vol. 145 (2010), 1865–1889.

——— "Middlemen: The Visible Market-Makers." *The Japanese Economic Review*, Vol. 69 (2018), 156–170.

——— "Middlemen: A Directed Search Equilibrium Approach." *The BE Journal of Macroeconomics (Advanced)*, Vol. 20 (2020), 1–37.

ZENNYO, Y. "Platform Encroachment and Own-content Bias." *The Journal of Industrial Economics*, Vol. 70 (2022), 684–710.

A Appendix

A.1 Proof of Lemma 1

The intersection point of $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$ is $(\lambda_0, c_0) = \left(\frac{k+\sqrt{k^2+4ku}}{2mu},\ u + k - \sqrt{k^2 + 4ku}\right)$. We can derive that

$$\frac{\partial b(\lambda, \mu)}{\partial \mu} = \frac{2\left(km\lambda + k - u(m\lambda)^2\right)}{(m\lambda\mu + m\lambda + \mu)^2},$$

which is positive if $\lambda < \lambda_0$ and negative if $\lambda > \lambda_0$. That is, as μ increases, $c = b(\lambda, \mu)$ rotates around (λ_0, c_0) clockwise, which implies that more suppliers with positive θ_F are selected (i.e. $q(\lambda, c, \mu)$ is increasing in μ for (λ, c) such that $\theta_F(\lambda, c) > 0$) and fewer suppliers with negative θ_F are selected (i.e. $q(\lambda, c, \mu)$ is decreasing in μ for (λ, c) such that $\theta_F(\lambda, c) < 0$).

Taking $\bar{c}(i)$ as given. If $c_0 \in [\underline{c}, \bar{c}(i)]$, since $g(\cdot)$ is everywhere positive in Ω, it holds that $\Theta(\mu, i) = \int_{\Omega(i)} q(\lambda, c, \mu)\theta_F(\lambda, c)dG$ is strictly increasing in μ.

If $c_0 > \bar{c}(i)$, and suppose $\lambda_0 < 1$, then there exist unique threshold values, denoted by $\underline{\mu} > 0$ and $\bar{\mu} \in (\underline{\mu}, \infty)$, such that the curve of $c = b(\lambda, \mu)$ lies entirely above $c = \bar{c}(i)$ for $\mu \in (\underline{\mu}, \bar{\mu})$, see Figure 12. For $\mu \in (\underline{\mu}, \bar{\mu})$, $\Theta = \int_{\Omega(i)} \theta_F(\lambda, c)dG$ is independent of μ, which means that μ does not influence the selection of suppliers. For $\mu \in (0, \underline{\mu}) \cup (\bar{\mu}, \infty)$, by the same logic as shown above, $\Theta(\mu, i)$ is strictly increasing in μ. Suppse $\lambda_0 \geq 1$, then $\Theta(\mu, i)$ is strictly increasing in μ for $\mu \in (0, \underline{\mu})$ and keeps constant for $\mu > \underline{\mu}$.

Common to all cases is that when μ approaches infinity, only suppliers with positive θ_F are selected, thus $\Theta(\infty, i) > 0$.

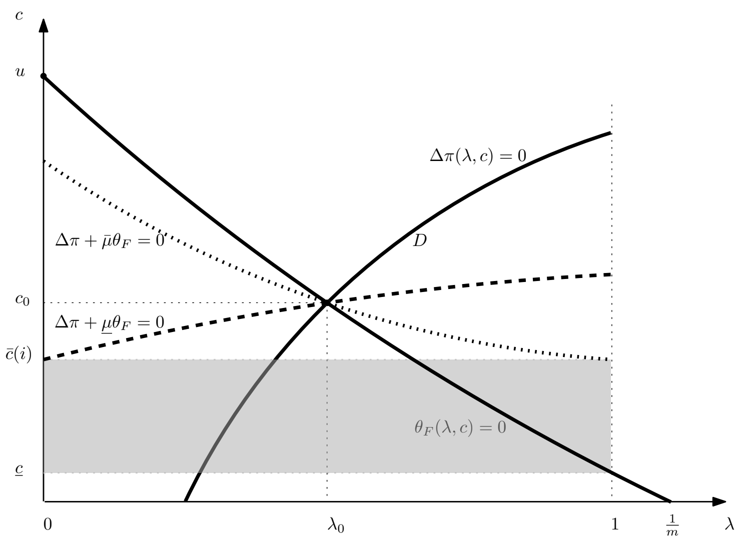


Figure 12: For $\mu \in (\underline{\mu}, \bar{\mu})$, $c = b(\lambda, \mu)$ lies above $\bar{c}$.

Now we show that μ is generically unique. Since $\Theta(\mu, i)$ is monotonically increasing in μ, if $\Theta(0, i) + L \geq 0$, then $\mu = 0$. If $\Theta(0, i) + L < 0$, then the liquidity constraint is binding at some $\mu \in (0, \infty)$, which is uniquely pinned down by $\Theta(\mu, i) + L = 0$. Note that when $L = -\int_{\Omega(i)} \theta_F(\lambda, c)dG \geq 0$ and $c_0 > \bar{c}(i)$, any $\mu \in [\underline{\mu}, \bar{\mu}]$ satisfies $\Theta(\mu, i) + L = 0$. In this case, we define the solution μ as $\underline{\mu}$. ■

A.2 Proof of Lemma 2, Theorem 1, Theorem 2 and Proposition 1

In text. ■

A.3 Proof of Corollary 1

Given that $\Theta(0,i) + L < 0$, $\mu(L,i)$ is determined by (15). Since $\Theta(\mu,i)$ is strictly increasing in μ outside the interval $[\underline{\mu}, \bar{\mu}]$ provided the range exists and also note that we have selected $\mu = \underline{\mu}$ if $\Theta(\underline{\mu}, i) = \Theta(\bar{\mu}, i) = L$, the statement follows. ■

A.4 Proof of Lemma 3

By the Euler equation (16), there are two cases. First, if $i \geq \mu(0,i)$, then $L = 0$. This case is valid either if $\mu(0,i) = 0$ (then $i > \mu = 0$ follows), or if $\mu(0,i) > 0$. Second, $i = \mu(L,i) > 0$ and $L > 0$, which requires that $\Theta(0,i) < 0$ and $i \leq \mu(0,i)$. ■

A.5 Proof of Corollary 2

It follows immediately from (17). ■

A.6 Proof of Lemma 4

Given that $b'_\lambda(\lambda,i) = \frac{2m(k+ik-i^2u)}{(i+m\lambda+im\lambda)^2}$, it is straightforward to verify that $b'_\lambda(\cdot) > 0$ if $i < i_0 \equiv \frac{k+\sqrt{k^2+4uk}}{2u}$, and $b'_\lambda(\cdot) < 0$ if $i > i_0$. The relationship between $c = b(\lambda, i)$ and $c = \bar{c}(i)$ can be obtained by comparing $b(1/m, i) = \frac{u-2k}{1+2i}$ with $\bar{c}(i) = \frac{1-i}{1+i}u$. If $i < i_0$, then $b(1/m,i) < \bar{c}(i)$. If $i > i_0$, then $b(1/m,i) > \bar{c}(i)$.■

A.7 Proof of Proposition 2

Formally, $\hat{B} = \left\{(\lambda, c) \in \Omega(i) \;\middle|\; \Delta\pi(\lambda,c) > 0, \theta_F(\lambda,c) < 0, c > c_0, \text{ and } -\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)} \geq i\right\}$, and $C = \left\{(\lambda, c) \in \Omega(i) \;\middle|\; \Delta\pi(\lambda,c) < 0, \theta_F(\lambda,c) > 0, c < c_0, \text{ and } -\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)} \leq i.\right\}$. Suppose $\hat{B}$ and C are non-empty. We need to show that, with a marginal decrease in m, suppliers in set $\hat{B}$ experience a lower $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)}$, and suppliers in set C experience a higher $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)}$. The latter has been proved in the text. So, let's consider some supplier in $\hat{B}$, denoted by (λ_1, c_1). Define $i_1 \equiv \frac{u-c_1}{u+c_1}$ so that $\bar{c}(i_1) = c_1$. A curve $c = b(\lambda, \mu)$ that passes through this supplier must have $\mu = \mu_1 \equiv \frac{\Delta\pi(\lambda_1,c_1)}{\theta_F(\lambda_1,c_1)}$. By Lemma 4, we have $c_1 = b(\lambda_1, \mu_1) < \bar{c}(\mu_1)$. Together with $\bar{c}(i_1) = c_1$, we have $\bar{c}(i_1) < \bar{c}(\mu_1)$. Then, $\frac{u-c_1}{u+c_1} = i_1 > \mu_1 = \frac{\Delta\pi(\lambda_1,c_1)}{\theta_F(\lambda_1,c_1)}$. Using (20) note that $\theta_F(\lambda_1,c_1) < 0$, we have $\frac{\partial \frac{\Delta\pi(\lambda_1,c_1)}{-\theta_F(\lambda_1,c_1)}}{\partial m} > 0$. Thus, suppliers in set $\hat{B}$ give a lower return $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)}$ to the middleman as m decreases. ■

A.8 Proof of Proposition 3

Note that $k < \bar{k}$ implies $c_0 > \underline{c}$. Let $\kappa \equiv \frac{k}{u}$ and $\bar{\kappa} \equiv \frac{\bar{k}}{u} \equiv \frac{(u-\underline{c})^2}{2u(u+\underline{c})}$. For all $i \le i_1$, the multiplier μ at $L = 0$ does not depend on i: $\mu(0,i) = \mu(0,0)$. We suppose the latter is positive throughout the proof, thus $\mu(i) = i$. A necessary condition of $\mu(0,0) > 0$ is $m > \tilde{m}(\kappa)$. We focus on the parameter space where $\kappa \in (0, \bar{\kappa})$, and $m \in (\tilde{m}(\kappa), 1]$ for each value of κ.

There are two cases for $\kappa < \bar{\kappa}$. One case is that $\kappa \le \underline{\kappa} \equiv \frac{(u-\bar{c})^2}{2u(u+\bar{c})} (< \bar{\kappa})$, then $c_0 \ge \bar{c}$. A marginal increase of i from $i = 0$ will select more suppliers into the finance contract and drop no suppliers from it. Thus, social welfare must be improved.

The other case is that $\kappa \in (\underline{\kappa}, \bar{\kappa})$, namely, $c_0 \in (\underline{c}, \bar{c})$. Then the middleman's selection rule for the finance contract is $q(\lambda, c, \mu) = 1$ iff $c \in [\underline{c}, b(\lambda, \mu)]$ whenever $b(\lambda, \mu) \ge \underline{c}$. Here, $b(\lambda, \mu) = \frac{m\lambda u - 2k + \mu(1-m\lambda)u}{m\lambda + \mu(1+m\lambda)}$. The welfare gain by having active middleman finance is

$$\Delta\mathcal{W}(\mu) = \int_{\lambda_l(\mu)}^{\lambda_h(\mu)} \int_{\underline{c}}^{b(\lambda,\mu)} \Big(m\lambda(u-c) - k\Big) g(\lambda, c) dc d\lambda + \int_{\lambda_h(\mu)}^{1} \int_{\underline{c}}^{\bar{c}} \Big(m\lambda(u-c) - k\Big) g(\lambda, c) dc d\lambda,$$

where we have imposed that $b(\lambda, \mu)$ is upward sloping with respect to λ, which is always the case when μ is in the neighborhood of $\mu = 0$. Here, $\lambda_h(\mu) = \min\{1, \frac{1}{m} \frac{2k - \mu(u-\bar{c})}{(u-\bar{c}) - \mu(u+\bar{c})}\}$, and $\lambda_l(\mu) = \max\{0, \frac{1}{m} \frac{2k - \mu(u-\underline{c})}{u - \underline{c} - \mu(u+\underline{c})}\}$.

Observe that $\frac{\partial \Delta\mathcal{W}(\mu)}{\partial \mu} = \int_{\lambda_l(\mu)}^{\lambda_h(\mu)} \Big(m\lambda(u - b(\lambda, \mu)) - k\Big) g(\lambda, b(\mu)) b'_\mu(\lambda, \mu) d\lambda$. Since (λ, c) follows a uniform distribution, g is a constant. Let $\propto$ represent "proportional to". Inserting $b(\lambda, 0) = u - \frac{2k}{m\lambda}$ and $b'(0) = 2u\left(\frac{\kappa}{m^2\lambda^2} + \frac{\kappa}{m\lambda} - 1\right)$, we have

$$\frac{\partial \Delta\mathcal{W}(\mu)}{\partial \mu}|_{\mu=0} \propto \int_{\lambda_l}^{\lambda_h} \Big[\frac{\kappa}{m^2\lambda^2} + \frac{\kappa}{m\lambda} - 1\Big] d\lambda, \tag{22}$$

where $\lambda_h \equiv \lambda_h(0) = \min\{1, \frac{1}{m} \frac{u\kappa}{(u-\bar{c})/2}\}$, and $\lambda_l \equiv \lambda_l(0) = \frac{1}{m} \frac{u\kappa}{(u-\underline{c})/2}$. Note that $\lambda_l < 1$. This is because with $m > \tilde{m}$, $c_\pi(1) > c_0 > \underline{c}$ where the second inequality is given by $\kappa < \bar{\kappa}$. And $c_\pi(1) > \underline{c}$ is equivalent to $\lambda_l < 1$. Define $\bar{\varepsilon} = \frac{u}{(u-\bar{c})/2}$ and $\underline{\varepsilon} = \frac{u}{(u-\underline{c})/2}$. It holds that $\bar{\varepsilon} > \underline{\varepsilon} > 2$. Furthermore, it is straightforward to check that $\underline{\kappa} < \frac{1}{\bar{\varepsilon}} < \bar{\kappa} < \frac{1}{\underline{\varepsilon}}$. With this, $\lambda_h = \min\{1, \frac{1}{m}\kappa\bar{\varepsilon}\}$, $\lambda_l = \frac{1}{m}\kappa\underline{\varepsilon}$.

Substitute for λ by $x \equiv m\lambda$. Then the upper and lower bounds become $x_h = \min\{m, \kappa\bar{\varepsilon}\}$, and $x_l = \kappa\underline{\varepsilon}$, and we have

$$\frac{\partial \Delta\mathcal{W}(\mu)}{\partial \mu}|_{\mu=0} \propto \int_{\kappa\underline{\varepsilon}}^{\min\{m, \kappa\bar{\varepsilon}\}} \Big[\frac{\kappa}{x^2} + \frac{\kappa}{x} - 1\Big] dx \equiv H(m, \kappa). \tag{23}$$

In the following, we look for the parameter space for $H(m, \kappa) > 0$.

Suppose $\bar{\varepsilon}\kappa < m$. Then, by (23) $H(m, \kappa)$ does not depend on m directly:

$$H(m, \kappa) = -\kappa(\bar{\varepsilon} - \underline{\varepsilon}) + \frac{\bar{\varepsilon} - \underline{\varepsilon}}{\bar{\varepsilon}\underline{\varepsilon}} + \kappa\Big(\log(\bar{\varepsilon}) - \log(\underline{\varepsilon})\Big),$$

which is positive iff $\kappa < \frac{1}{\bar{\varepsilon}\underline{\varepsilon}} \frac{1}{1 - \frac{\log(\bar{\varepsilon}) - \log(\underline{\varepsilon})}{\bar{\varepsilon} - \underline{\varepsilon}}} \equiv \kappa^*$.

Suppose $\bar{\varepsilon}\kappa \geq m$, then

$$H(m,\kappa) = \int_{\kappa\underline{\varepsilon}}^{m} \left[\frac{\kappa}{x^2} + \frac{\kappa}{x} - 1\right] dx.$$

In this case, $H(m,\kappa)$ has the following properties:

- $H(m,\kappa)$ is strictly decreasing in m for $m \in [\tilde{m}, 1]$ since $\frac{\partial H(m,\kappa)}{\partial m} = \frac{\kappa + \kappa m - m^2}{m^2} < 0$ for $m > \tilde{m}$.
- $H(\tilde{m}(\kappa),\kappa) \geq 0$ since $\frac{\kappa}{x^2} + \frac{\kappa}{x} - 1 \geq 0$ for $x \leq \tilde{m}$. $H(\tilde{m}(\kappa),\kappa) = 0$ only if $\kappa = \bar{\kappa}$.
- $H(1,\kappa) < 0$. To see this, using (23)

$$H(1,\kappa) = \Big[-\lambda + \kappa\Big(\log(\lambda) - 1/\lambda\Big)\Big]_{\underline{\lambda}}^{1} = (\underline{\varepsilon} - 1)(\kappa - \frac{1}{\underline{\varepsilon}}) - \kappa \log(\kappa\underline{\varepsilon}) \equiv h(\kappa).$$

  Note that $h'(\kappa) = \underline{\varepsilon} - 2 - \log(\underline{\varepsilon}\kappa) > 0$ since $\underline{\varepsilon} > 2$ and $\kappa\underline{\varepsilon} = \underline{\lambda} < 1$. Then $h(\kappa) < h(\bar{\kappa}) < h(1/\underline{\varepsilon}) = 0$ (since $\bar{\kappa} < \frac{1}{\underline{\varepsilon}}$). Thus, $H(1,\kappa) < 0$.

These properties together imply that there must exist $m^*(\kappa) \in [\tilde{m}(\kappa), 1)$ such that $H(m,\kappa) > 0$ iff $m < m^*(\kappa)$.

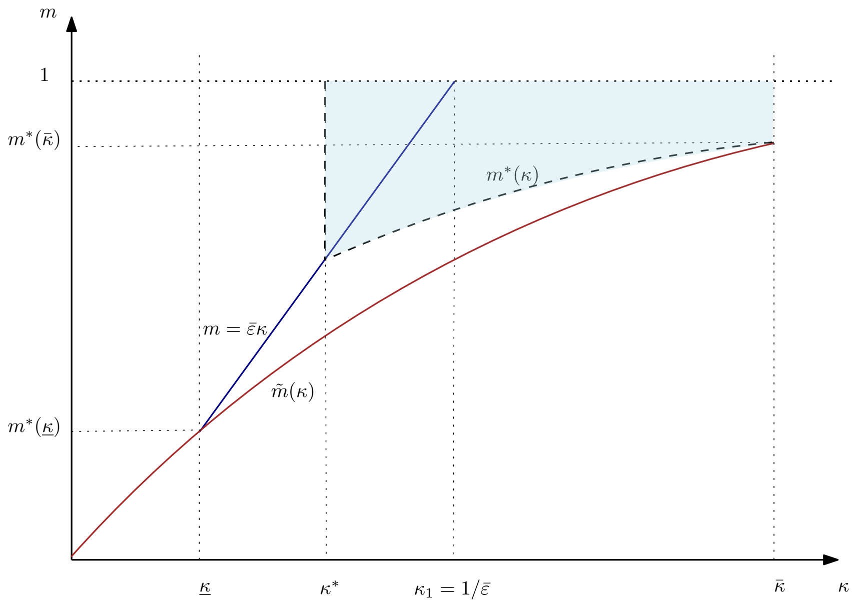


Figure 13: The parameter space that determines the sign of $H(m,\kappa)$ (in shaded region $H(\cdot) < 0$.)

Figure 13 illustrates the parameter space of (κ, m) where $m = \tilde{m}(\kappa)$ (upward concave curve) and $m = \bar{\varepsilon}\kappa$ (upward straight line) are plotted. As argued above, we shall focus on $m > \tilde{m}(\kappa)$ since only in that case it is plausible that $\mu(0,0) > 0$. Whether m larger or smaller than $\bar{\varepsilon}\kappa$ (the upward straight line) determines the upper bound of the integral in $H(m,\kappa)$.

In this figure, we have applied the following properties of $m = \tilde{m}(\kappa)$: (1) $\tilde{m}(0) = 0$, and $\tilde{m}(\bar{\kappa}) < 1$; (2) It is increasing and concave on $\kappa \in [0, \bar{\kappa}]$: $\tilde{m}'(\kappa) = \frac{1}{2}\left(1 + \frac{\kappa+2}{\sqrt{\kappa^2+4\kappa}}\right) > 0$, $\tilde{m}''(\kappa) = -\frac{2}{(4\kappa+\kappa^2)^{3/2}} < 0$. We also applied the following properties of $m = \bar{\varepsilon}\kappa$: $\bar{\varepsilon}\bar{\kappa} > 1$, and $\bar{\varepsilon}\underline{\kappa} = \tilde{m}(\underline{\kappa})$.

First consider the case of $m > \bar{\varepsilon}\kappa$, then $H(m,\kappa) > 0$ iff $\kappa < \kappa^*$. Define $\kappa_1 = \frac{1}{\bar{\varepsilon}}$ the value of κ where $m = \bar{\varepsilon}\kappa$ intersects with $m = 1$. κ^* must lie between $\underline{\kappa}$ and κ_1. $\kappa^* > \underline{\kappa}$ because $H(1,\underline{\kappa}) > 0$ (when $\kappa = \underline{\kappa}$, we have $c_0 = \bar{c}$ and a marginal increase in i only selected more suppliers into the finance contract without dropping suppliers out of it.) $\kappa^* < \kappa_1$ because $H(1,\kappa_1) < 0$.

Second, consider the case where $m < \bar{\varepsilon}\kappa$. In this scenario, $H(m,\kappa) > 0$ if and only if $m < m^*(\kappa)$. As shown in the figure, $m^*(\kappa)$ connects $(\bar{\kappa}, \tilde{m}(\bar{\kappa}))$ at one end and $(\kappa^*, \bar{\varepsilon}\kappa^*)$ at the other end. We have $m^*(\bar{\kappa}) = \tilde{m}(\bar{\kappa})$ because when $\kappa = \bar{\kappa}$ and $m = \tilde{m}(\bar{\kappa})$, $\Delta\mathcal{W}(\mu)$ is constantly zero and independent of μ. By definition of κ^*, $H(m^*(\kappa^*),\kappa^*) = 0$. ■

A.9 Proof of Proposition 4

Let $\Pi(i^s, i) \equiv \int_{\tilde{\Omega}(i^s)} q(\lambda, c)\Delta\pi(\lambda, c)dG$ be the maximized profits of the middleman from activating the finance service taking nominal interest rate $i < i_1$ as given. Let $c_{\Delta\pi}(\lambda) = u - \frac{2k}{m\lambda}$ denote the curve of (λ, c) such that $\Delta\pi(\lambda, c) = 0$. It can be shown that $c^s(\lambda, i^s)$ and $c_{\Delta\pi}(\lambda)$ cross each other at most once.

If $c^s(1, i^s) > c_{\Delta\pi}(1)$, or equivalently, $i^s < \frac{k}{mu-2k}$, then $c^s(\lambda, i^s) > c_{\Delta\pi}(\lambda)$ for all $\lambda \in [0,1]$, meaning that all suppliers with positive profits $\Delta\pi(\lambda, c)$ are excluded from $\tilde{\Omega}(i^s)$. Thus, we must have $\Pi(i^s, i) = 0$. On the other hand, if $i^s \geq \bar{i}^s \equiv \frac{u-\underline{c}}{2\underline{c}}$, then $\tilde{\Omega}(i^s) = \Omega$, resulting in $\Pi(i^s, i) > 0$. Note that $\lambda_0 < 1$ implies $c_{\Delta\pi}(1) > \underline{c}$, which is equivalent to $\bar{i}^s > \frac{k}{mu-2k}$.

Finally, $\Pi(\cdot)$ is weakly increasing in i^s, because as i^s increases, the set of feasible suppliers $\tilde{\Omega}(i^s)$ becomes larger. Therefore, $\underline{i}^s \in [\frac{k}{mu-2k}, \bar{i}^s)$ must exist. Combined with the suppliers' money-holding decision rule (see condition (21) in the main text), this proves the claims in the proposition. ■

A.10 Proof of Proposition 5

For $i \in (0, i_2)$, it holds that $c = c^s(\lambda, i)$ must cross $c = \bar{c}(i)$ since $c^s(0,i) < \bar{c}(i) < c^s(1,i)$. Thus, there exist suppliers with (λ, c) satisfying $c \in [\underline{c}, \min\{\bar{c}(i), c^s(\lambda, i)\}]$. In equilibrium, these suppliers choose to hold money by themselves.

Turn to the middleman's problem. The feasible set of suppliers is given by $\{(\lambda, c) \in \Omega | c^s(\lambda, i) \leq c \leq \bar{c}(i)\}$. Note that $\bar{c}(i)$ and $c^s(\lambda, i)$ intersect at $(\lambda_1, c_1) = \left(1 - i, \frac{1-i}{1+i}u\right)$. In Figure 14, the feasible set of suppliers is represented by the green region between $\bar{c}(i)$ and $c^s(\lambda, i)$ and to the left of (λ_1, c_1). A necessary condition for active middleman finance in equilibrium is that (λ_1, c_1) locates below the curve of $\Delta\pi(\lambda, c) = 0$, i.e., there exists some set of i such that

$$\Delta\pi(\lambda_1, c_1) = \Delta\pi\left(1 - i, \frac{1-i}{1+i}u\right) > 0. \tag{24}$$

Otherwise, all suppliers in the feasible set give a negative $\Delta\pi(\cdot)$ to the middleman. (24) gives $i \in (i_-, i_+)$, where $i_- \equiv \frac{-\sqrt{k^2-6ku+u^2}-k+u}{2u}$ and $i_+ = \frac{\sqrt{k^2-6ku+u^2}-k+u}{2u}$. In Figure 15, we illustrate $c = c^s(\lambda, i)$ and $c = \bar{c}(i)$ at the two levels of nominal interest rates, i_- and i_+, as well as the

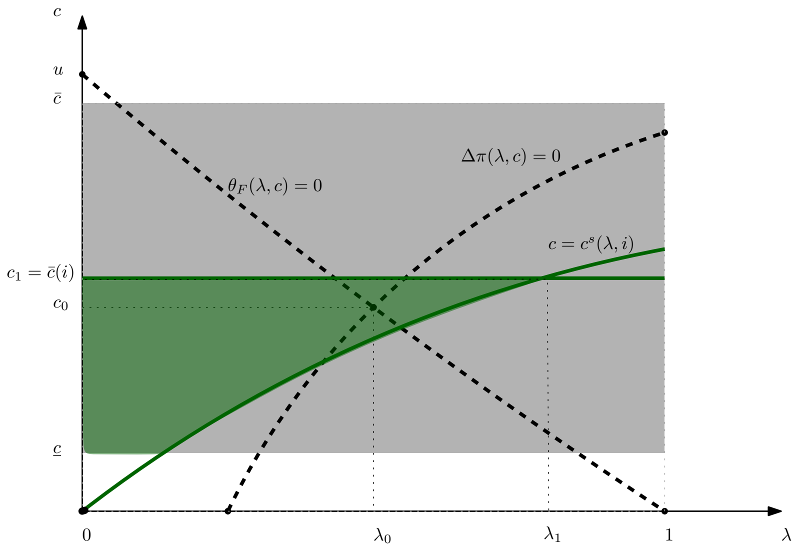


Figure 14: Illustration for $\pi(\lambda_1, c_1) < 0$

feasible set of suppliers in each case, in green and red, respectively. For i_- and i_+ to exist, the middleman needs to be sufficiently efficient, i.e., $k < (3 - 2\sqrt{2})u$.

Define i_0 by $\bar{c}(i_0) = c_0$. Note that $i_- < i_0$ always holds, and $i_+ > i_0$ if and only if $k < u/6 \left(< (3 - 2\sqrt{2})u\right)$. Also note that $c = c^s(\lambda, i)$ crosses $c = c_\pi(\lambda)$ at $c = k/i$. We can guarantee that the middleman providing finance is profitable if $c = c^s(\lambda, i)$ crosses $c = c_\pi(\lambda)$ below c_0, which holds if $i > k/c_0$. Moreover, we need to make sure such i to be in (i_-, i_+), which requires $i_+ > i_0$, or equivalently $k < u/6$. To sum up, under $k < u/6$ and $i > k/c_0$, there exists a set of suppliers who contribute positive $\Delta\pi$ and positive θ_F, represented by the green region in Figure 16. As a result, the middleman finance is profitable and active in equilibrium. ■

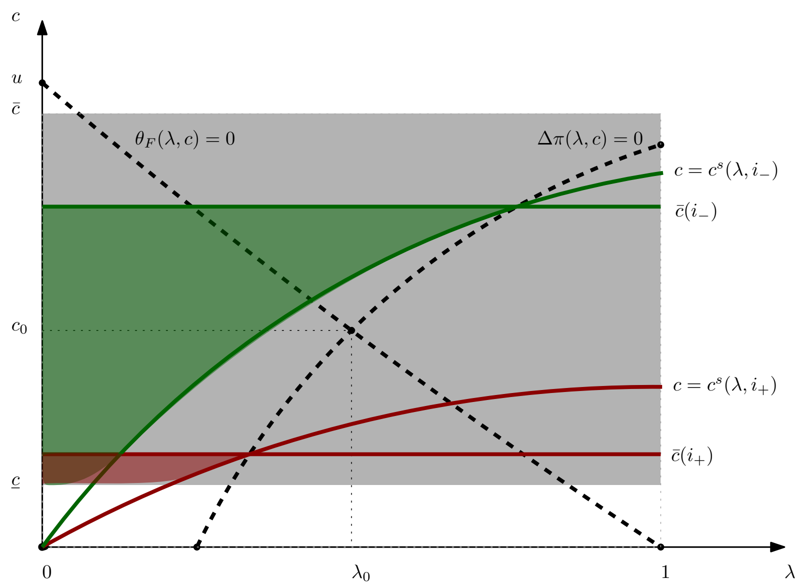


Figure 15: Illustration for i_- and i_+

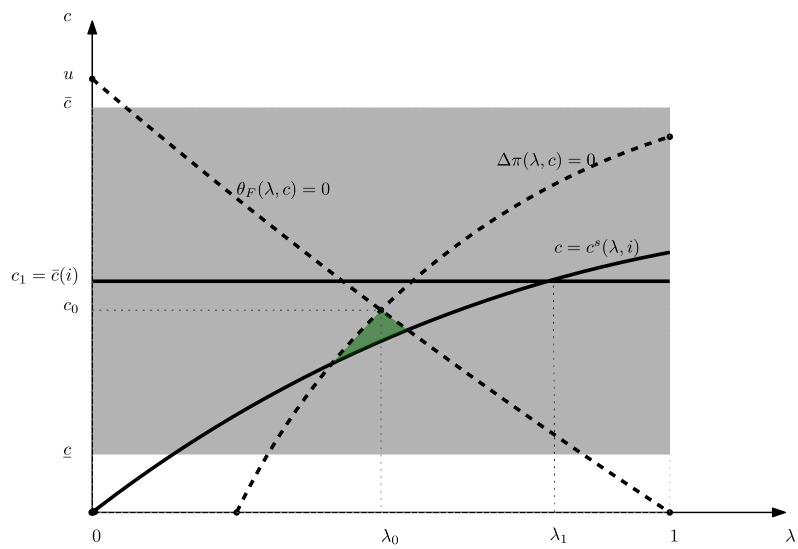


Figure 16: Illustration for $k/i < c_0$