A Model of Supply Chain Finance

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What is Supply Chain Finance (SCF)?

Consider a large buyer firm, e.g., Walmart, Siemens, etc.

- a large number of heterogeneous suppliers
- suppliers face shortage of working capital from time to time

Supply chain finance:

- a program offered by the buyer firm (possibly with financiers)
- select among its suppliers to join
- suppliers are given extended payment terms
- suppliers can request immediate payment at a small discount

The Co-op Partners with PrimeRevenue to Protect Suppliers Amid Economic Volatility

UK's sixth largest food retailer makes strategic transition to PrimeRevenue platform

Atlanta, GA – Manchester, UK, August 11, 2020 – PrimeRevenue, the leading platform for working capital finance solutions, and The Co-operative Group, today announce a new supply chain finance partnership. Barclays Bank PLC, who introduced The Co-op to PrimeRevenue, will be providing funding on the supply chain finance programme followed by other financial institutions as the programme grows.

Co-op has made the strategic decision to partner with PrimeRevenue for its new supply chain finance offering. Fueled by a highly challenging business climate heightened by the pandemic, the company aims to offer suppliers a simple method of early payment to help with their cash flow without having a detrimental impact to Co-op's own cash position. This is particularly relevant in the current environment where the old adage "cash is king" has never been truer.

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Why do we care?

SCF is happening and on the rise

- SCF has been widely adopted by large corporations
- The size of the SCF market is $1.8 trillion globally in 2021

SCF can be a "SLEEPING RISK" that "MASKS EPISODES OF FINANCIAL STRESS." (S&P Global Inc.)

- The buyer firm (and associated supply chain) may face difficulties if financing cost increases
- FASB: Starting in 2023, corporations will have to disclose the terms and size of the SCF programs in the financial statement.

Preview of the model

- A simple model of a middleman funding suppliers.
- Heterogeneous suppliers: productivity and liquidity needs.
- The middleman selects suppliers into the SCF program
- We then integrate this model into a standard monetary framework (Lagos and Wright, 2005).

Preview of key results

Liquidity cross-subsidization

- use liquidity from suppliers with negative profits
- to fund suppliers with positive profits
- links to the cost of market liquidity

Friedman rule can be suboptimal

- market liquidity is more costly $\Rightarrow$ SCF replies more on suppliers' liquidity
- more suppliers included $\Rightarrow$ more trade created (under some conditions)

Related literature

- Multi-product intermediaries:
  - Rhodes et al. (2021), Spulber (1996).
  - Liquidity issues are not addressed
- Banking and Money
  - Berentsen et al. (2007), Gu et al. (2013), Andolfatto et al. (2019)
  - Our model emphasizes the ex-ante section of depositors
  - Unlike in Diamond and Dybvig (1983), the late-type depositors in our model do not have the incentive to run
- Supply chain finance:
  - In econ and finance, closely related is trade credit.
  - In management science, e.g., Kouvelis and Xu (2021)
  - Our model: one big buyer firm with many suppliers.

This talk

1. Benchmark model
   - an endowment economy (one-period model)
   - a subperiod (DM) in Lagos and Wright (2015) framework
2. Endogenous liquidity holdings
3. Welfare analysis
4. Extension

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1. The Benchmark Model

Agents

- A mass of suppliers:
  - each produces a unique and indivisible good
  - constant marginal costs, $c \in [\underline{c}, \bar{c}]$, differ among suppliers
  - c is publicly observable
- A mass of consumers:
  - unit demand for each good with *common* utility $u > \bar{c}$
- One middleman:
  - purchases from suppliers and resells to consumers
  - operates an SCF program (specified later)
  - fixed cost $k > 0$ to handle each supplier

Endowments/Liquidity

- There is a *numeraire* good (money)
- Consumers have enough endowment of numeraire
- The middleman has endowment (measure) $L \geq 0$
- Suppliers have no endowment, however, production cost c must be paid using the numeraire good.

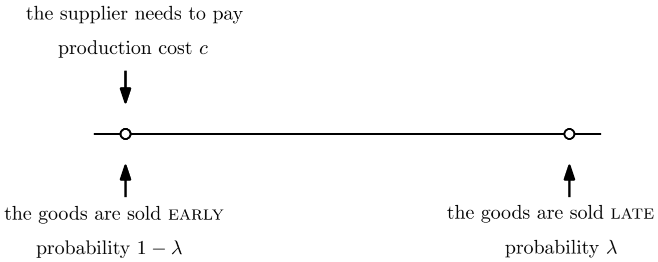
Retail market

- Without the middleman, suppliers can trade directly with consumers.
- Suppliers can meet ALL consumers, trade bilaterally:
  - if a trade occurs, the retail-trade surplus is split equally:

$$p - c = (u - c)/2$$

  - however, trade may not occur due to liquidity frictions.

Liquidity shocks



- A liquidity shock is realized at the beginning of period
- With prob $1 - \lambda$, a supplier encounters no liquidity issue, c can be covered by using retail revenue
- With prob λ, a supplier encounters a liquidity issue, the supplier cannot produce since he has no numeraire

Ex ante heterogeneity of suppliers

- Each supplier is indexed by

$$(\lambda, c) \in \Omega = [0, 1] \times [\underline{c}, \bar{c}],$$

  where λ is prob liquidity shock, c is the const marginal cost

- (λ, c) is publicly observable, following a distribution C.D.F. G, P.D.F. $g > 0$ on Ω
- The realization of the liquidity shock can be public or private information.

Middleman and SCF program

- Middleman observes (λ, c), and selects suppliers into SCF program.
- Selection policy:

$$q(\lambda, c) = \begin{cases} 1 & \text{if } (\lambda, c) \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

Middleman and SCF program (cont.)

Given a supplier is invited $q(\lambda, c) = 1$, the middleman gives a TIOLI offer based on (λ, c):

- The middleman sells the goods on behalf of the supplier
  - bilateral trade / suppliers quite the market.
- The middleman transfers a revenue $f(\lambda, c)$ at end of period.
- The middleman pays c to the supplier at beginning of period.

An SCF program can be represented by:

$$\{q(\lambda, c), f(\lambda, c)\}_{(\lambda,c)\in\Omega} \in \{0, 1\} \times \mathbb{R}_+.$$

SCF program (alternative setting)

Intermediary, instead of a middleman

Given $q(\lambda, c) = 1$, SCF gives a TIOLI offer based on (λ, c):

- Supplier gives his retail revenue p to the intermediary
- The intermediary transfers to the supplier a reward at end of period
  - $f^E(\lambda, c)$ if revenue transferred at beginning of period
  - $f^L(\lambda, c)$ if revenue transferred at end of period
- The intermediary always pays c to supplier at beginning of period

An SCF program can be represented by:

$$\{q(\lambda, c), f^E(\lambda, c), f^L(\lambda, c)\}_{(\lambda, c)\in\Omega} \in \{0, 1\} \times \mathbb{R}_+.$$

Timing

1. Middleman announces SCF, and invites suppliers.
   Suppliers decide to accept or not.

2. Liquidity shock of each supplier is realized, suppliers produce.
   Middleman pays c to participating suppliers, meanwhile, trade occurs in the retail market.

3. The middleman pays each supplier $f(\lambda, c)$.

Analysis

Solution concept

- Complete information game
- Subgame perfection

Suppliers' participation decision

- Supplier (λ, c) joins SCF program if

$$\underbrace{f(\lambda, c)}_{\textit{join SCF}} \geq \underbrace{(1-\lambda)(u-c)/2}_{\textit{not join SCF}}$$
$$\Rightarrow f(\lambda, c) = (1-\lambda)(u-c)/2$$

Profits and liquidity contributions to SCF

- A supplier contributes to SCF in PROFIT and LIQUIDITY.
- Profit contribution:

$$\pi(\lambda, c) = p - c - f - k = \lambda(u - c)/2 - k.$$

- Liquidity contribution at the time of production:

$$\theta(\lambda, c) = (1 - \lambda)p - c = (1 - \lambda)(u + c)/2 - c.$$

- π and θ can be positive or negative depending on (λ, c)

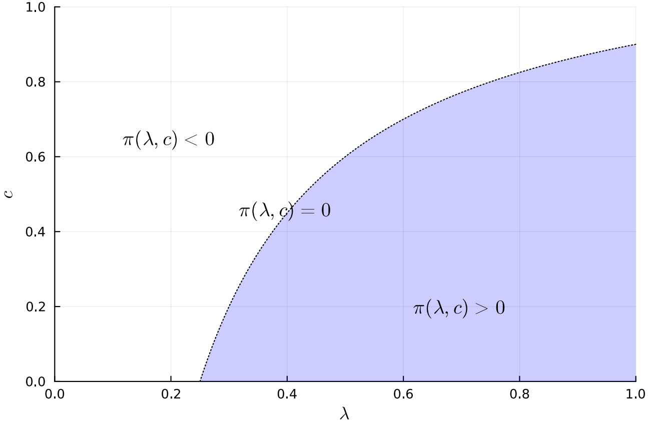


Figure: profit contributions in (λ, c) space

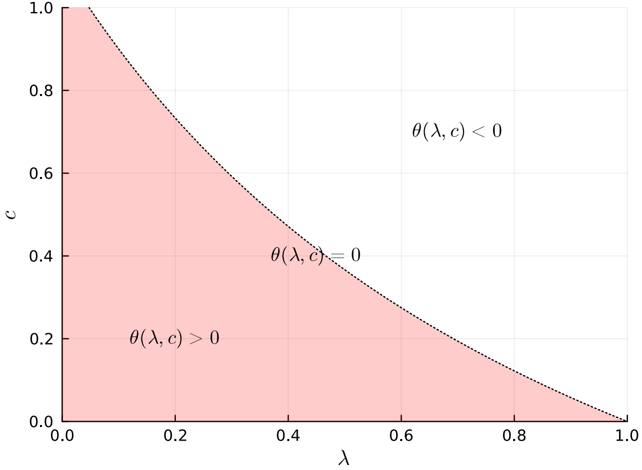


Figure: liquidity contributions in (λ, c) space

- The middleman's profit maximization problem:

$$\max_{q(\lambda,c)\in\{0,1\}} \int_{\Omega} q(\lambda,c)\pi(\lambda,c)dG$$

subject to the liquidity constraint:

$$\underbrace{\int_{\Omega} q(\lambda,c)\theta(\lambda,c)dG}_{\text{total liquidity}} + L \geq 0,$$

where initial liquidity holdings $L \geq 0$ (exogenous for now).

Profit-maximizing selection policy

- The middleman's problem can be solved using the Lagrangian:

$$\mathcal{L} = \int_{\Omega} q(\lambda, c)\Big[\pi(\lambda, c) + \mu\theta(\lambda, c)\Big] dG(\lambda, c).$$

- $\mu \geq 0$: Lagrangian multiplier of the liquidity constraint; the shadow value of liquidity.

- The optimal selection policy:

$$q(\lambda, c, \mu) = \begin{cases} 1 & \text{if } \pi(\lambda, c) + \mu\theta(\lambda, c) \geq 0 \\ 0 & \text{if otherwise.} \end{cases}$$

Proposition (Liquidity cross-subsidization)

The middleman optimally selects suppliers from

- *Region A: positive profit and positive liquidity contributions*

$$\pi(\lambda, c) \geq 0, \quad \theta(\lambda, c) \geq 0$$

- *Region B: positive profit and negative liquidity*

$$\pi(\lambda, c) > 0, \quad \theta(\lambda, c) < 0, \quad \underbrace{\pi/(-\theta)}_{\textit{returns}} \geq \mu$$

- *Region C: negative profit and positive liquidity*

$$\pi(\lambda, c) < 0, \quad \theta(\lambda, c) > 0, \quad \underbrace{-\pi/\theta}_{\textit{costs}} \leq \mu$$

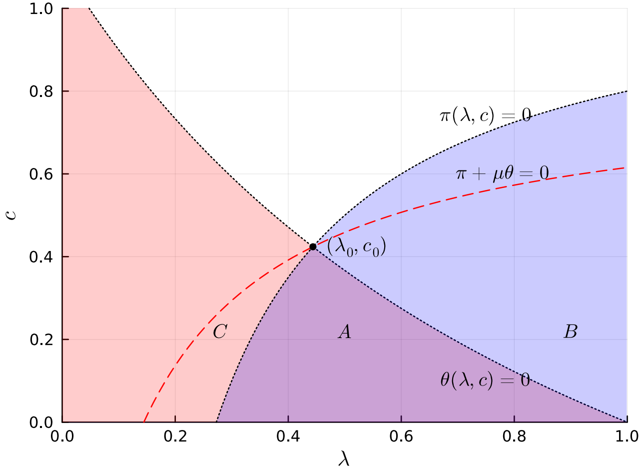


Figure: Liquidity cross-subsidization

Determine μ

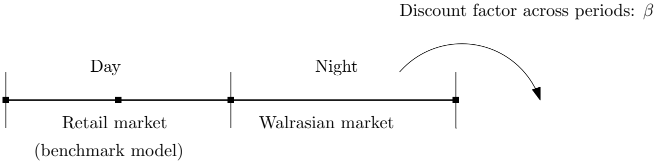
The liquidity constraint determines $\mu = \mu(L)$:

$$\int_\Omega q(\lambda, c, \mu)\theta(\lambda, c)dG + L = 0.$$

- $\mu(L) = 0$: liquidity does not matter for selecting suppliers; selection is solely based on $\pi(\lambda, c)$
- $\mu(L) > 0$: liquidity cross-subsidization, strictly decreases in L
- $\mu(0)$: the liquidity value at $L = 0$, or shadow price of the first marginal unit of liquidity

2. Endogenous liquidity holdings

Standard monetary approach (Lagos and Wright, 2005)



- Day market (the benchmark model)
  - the numeraire good is a medium of exchange, e.g., fiat money
  - suppliers must pay for production costs using fiat money
- Night market (Walrasian)
  - all other markets, where the middleman and consumers can "earn" fiat money by producing a "general good"
  - 1 unit of fiat money worth ϕ_t units of general good: $L_t = \phi_t l_t$.

Liquidity holdings of the middleman

- The middleman chooses $l(\equiv L/\phi)$ units fiat money

$$\max_{l\geq 0} \Big\{ -\phi_{t-1} l + \beta V_t(l) \Big\} \quad \Rightarrow \quad \phi_{t-1} \geq \beta V_t'(l).$$

- middleman's value of carrying l units of fiat money:

$$V_t(l) = \Big\{ \phi_t l + \max_{q(\lambda,c)} \int_\Omega q(\lambda, c)\pi(\lambda, c) dG, \quad \text{s.t.} \;\; \Theta + \phi_t l \geq 0. \Big\}$$

$$\Rightarrow \;\; V_t'(l) = \phi_t \Big(1 + \mu(L) \Big)$$

- Euler equation: $\phi_{t+1} \geq \beta \phi_t (1 + \mu(L))$, or equivalently

$$i \geq \mu(L).$$

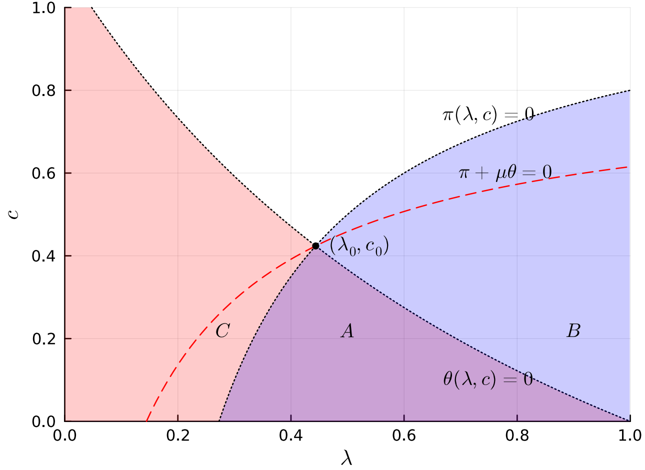
Proposition

For $i \leq \bar{i}$, there exists a unique monetary equilibrium with SCF program described by $q(\lambda, c, \mu)$, $f(\lambda, c)$, shadow value of liquidity:

$$\mu = \min\{\mu(0), i\},$$

and middleman's liquidity holding:

$$\begin{cases} \mu(L^*) = i & \text{if } \ i < \mu(0); \\ L^* = 0 & \text{if } \ i \geq \mu(0). \end{cases}$$



In equilibrium, $\mu = \min\{\mu(0), i\}$.

3. Welfare and Inflation

Planner's problem

- Trade surplus:

$$v(\lambda, c) = \lambda(u - c) - k.$$

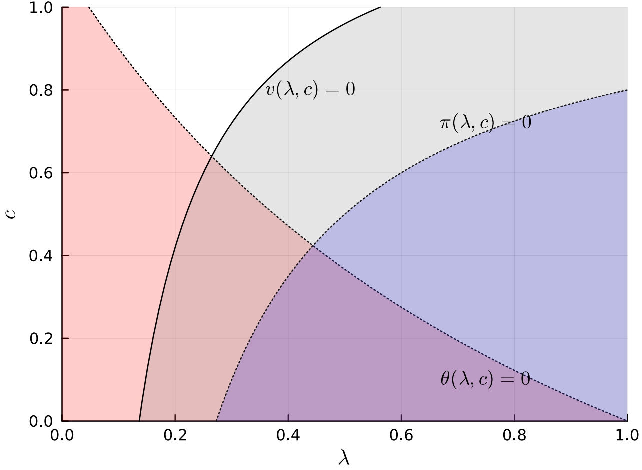
Rather than profits $\pi(\lambda, c) = \lambda(u - c)/2 - k$.

- Planner's problem:

$$\max_{I(\lambda,c)} \int_{\Omega} I(\lambda, c) v(\lambda, c) dG.$$

- The efficient allocation:

$$I(\lambda, c) = 1 \text{ if } v(\lambda, c) \geq 0$$



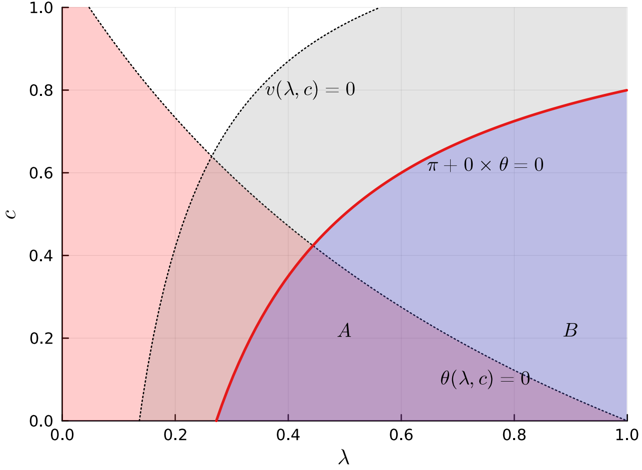
$v(\lambda, c) = 0$
$\pi(\lambda, c) = 0$
$\theta(\lambda, c) = 0$
1.0
0.8
0.6
0.4
0.2
0.0
c
0.0
0.2
0.4
0.6
0.8
1.0
λ

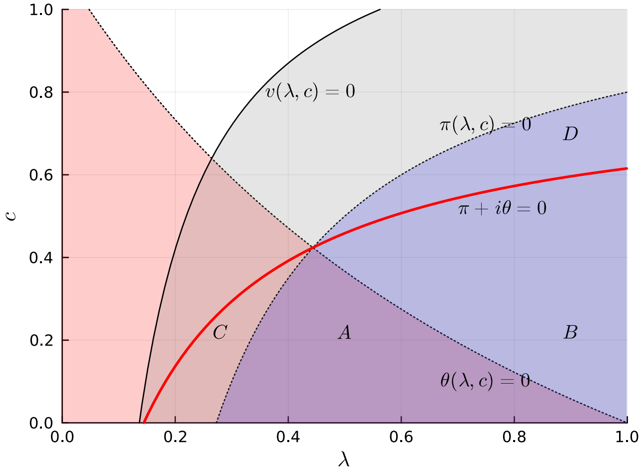
SCF is welfare improving

- At any given $i \leq \bar{i}$, SCF leads to an increase in welfare:

$$\begin{aligned}\Delta\mathcal{W}(i) &= \int_{\Omega} q(\lambda, c, \mu(i))v(\lambda, c)dG \\ &\geq \int_{\Omega} q(\lambda, c, \mu(i))\pi(\lambda, c)dG > 0.\end{aligned}$$

- $\Delta\mathcal{W}(i)$ may increase in i:
  - $k > 0$; not all suppliers are in SCF under $i = 0$
  - higher i induces more cross-subsidization and more trade


$v(\lambda, c) = 0$
$\pi + 0 \times \theta = 0$
A
B
$\theta(\lambda, c) = 0$
c
λ
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
c
λ
$v(\lambda, c) = 0$
$\pi(\lambda, c) = 0$
$\pi + i\theta = 0$
$\theta(\lambda, c) = 0$
A
B
C
D

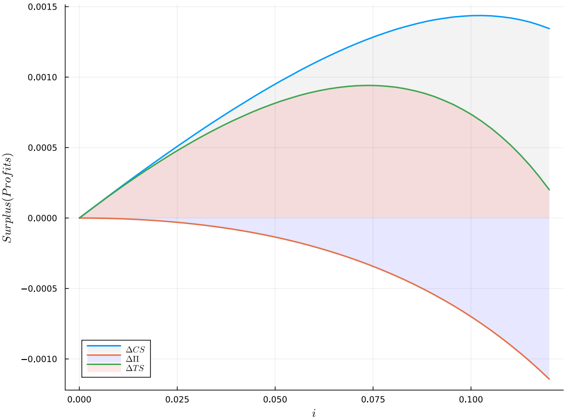


Figure: Welfare is non-monotonic in i under uniform distribution of (λ, c)

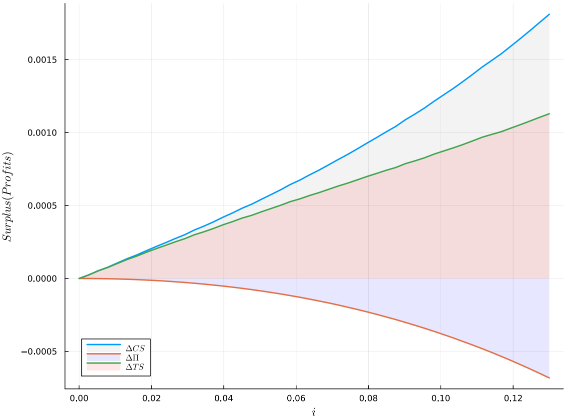


Figure: Welfare increases in i under Beta distributions of λ and c

4. Extension: If suppliers access to market liquidity?

Suppliers' money holding

- A supplier needs to hold $\hat{m} = \frac{c}{\phi_{+1}}$ in the previous night market:

$$\text{cost}: \ \phi \hat{m} \ \text{ v.s. } \ \text{benefit}: \ \beta^s \Big[\phi_{+1}\hat{m} + \lambda(p - c)\Big].$$

- Suppliers purchase money in previous night market if

$$c < c^s(\lambda, i) \equiv \frac{\lambda}{i^s + \lambda} p.$$

- The *updated* selection rule:

$$q(\lambda, c, \mu) = 1 \ \text{ if } \ \pi(\lambda, c) + \mu\theta(\lambda, c) \geq 0 \ \text{ and } \ c \geq c^s(\lambda, i).$$

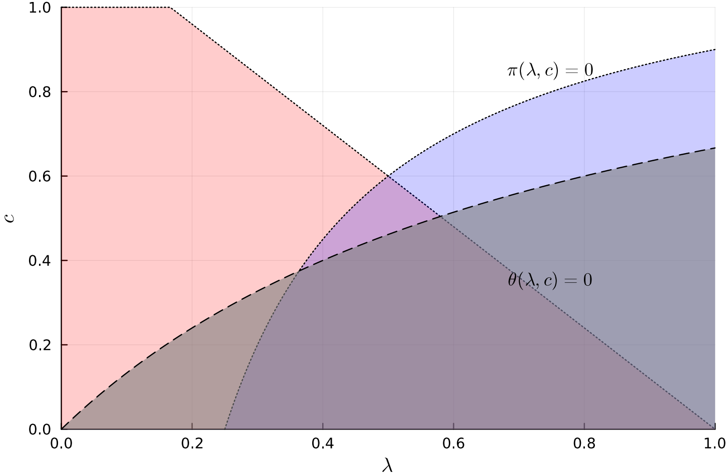


Figure: Suppliers access to liquidity (high i)

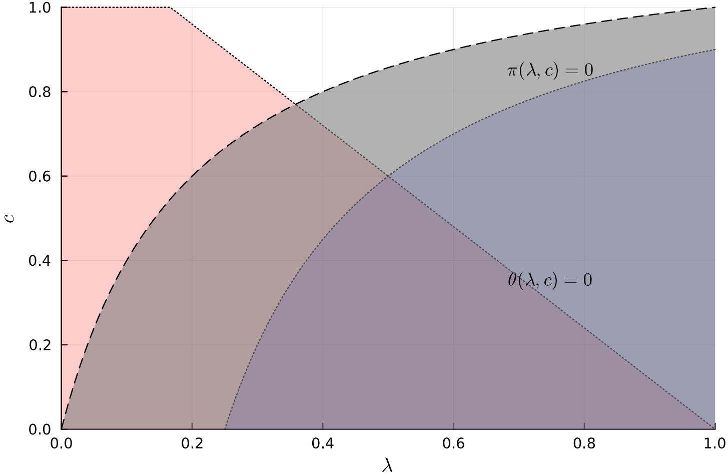
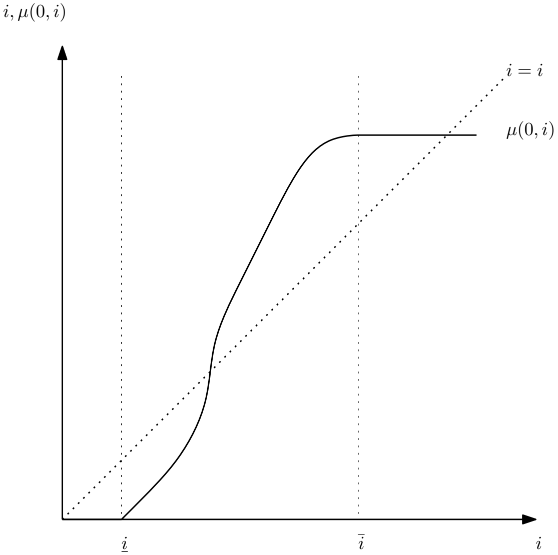


Figure: Suppliers access to liquidity (low i)


$i, \mu(0,i)$
$i = i$
$\mu(0,i)$
$\underline{i}$
$\bar{i}$
i

Takeaways

- SCF: a middleman pools liquidity from (early) suppliers, and funds suppliers for liquidity needs
- SCF features LIQUIDITY CROSS-SUBSIDIZATION
- SCF helps mitigate the high cost of market liquidity
- Deviating from Friedman rule can be welfare-enhancing