Marketmaking Middlemen*

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Abstract

This paper develops a model in which market structure is determined endogenously by the choice of intermediation mode. There are two representative modes of intermediation that are widely used in real-life markets: one is a middleman mode where an intermediary holds inventories which he stocks from the wholesale market for the purpose of reselling to buyers; the other is a market-making mode where an intermediary offers a platform for buyers and sellers to trade with each other. We show that a *marketmaking middleman*, who adopts the mixture of these two intermediation modes, can emerge in a directed search equilibrium and discuss the implications of this on the market structure. Our main insight survives with competing intermediaries.

Keywords: Middlemen, Marketmakers, Platform, Directed Search

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1 Introduction

This paper develops a framework in which market structure is determined by the intermediation service offered to customers. There are two representative modes of intermediation that are widely used in real-life markets. In one mode, an intermediary acts as a *middleman* (or a *merchant*), who is specialized in buying and selling for his own account and typically operates with inventory holdings (e.g. supermarkets, traditional brick and mortar retailers, and dealers in financial and steel markets). In the other mode, an intermediary acts as a *marketmaker*, who offers a marketplace for fees, where the participating buyers and sellers can search and trade with each other and at least one side of the market pays a fee for using the platform (e.g. auction sites, brokers in goods or financial markets, and many real estate agencies).

The market-making mode became more appropriate since new advanced internet technology facilitated the use of online platforms in the late 1990s and early 2000s. In financial markets, an expanded platform sector is adopted in a specialist market, i.e., the New York Stock Exchange (NYSE),[1] and even in a typical dealers' (i.e., middlemen's) market, i.e the NASDAQ. In goods and service markets, the electronic retailer Amazon.com and the online hotel/travel reservation agency Expedia.com, who have been a pure middleman, also act as a marketmaker, by allowing other suppliers to participate in their platform as independent sellers. In housing markets, some entrepreneurs run a dealer company (developing and owning luxury apartments and residential towers) and a brokerage company simultaneously in the same market.

Common to all the above examples is that intermediaries operate both as a middleman and a marketmaker at the same time. This is what we call a *marketmaking middleman*. Hence, the first puzzle is to explain the emergence of marketmaking middlemen, i.e., why the middleman or the platform sector has not become the exclusive avenue of trade, despite the recent technological advancements.

We also observe considerable differences in the microstructure of trade in these markets. The NASDAQ is still a more 'middlemen-based' market relative to the NYSE. While some intermediaries in housing markets are marketmaking middlemen, many intermediaries are brokers. Other online intermediaries, such as eBay and Booking.com, are pure marketmakers, who do not buy

[1] In the finance literature, the following terminologies are used to classify intermediaries: brokers refer to intermediaries who do not trade for their own accounts, but act merely as conduits for customer orders, akin to our marketmakers; dealers refer to intermediaries who do trade for their own accounts, akin to our middlemen/merchants. The marketmakers (or specialists) in financial markets quote prices to buy or sell assets as well as take market positions, so they may correspond broadly to our market-making middlemen.

and sell on their own accounts, like Amazon.com and Expedia.com do. They solely concentrate on their platform business. So the second puzzle is to explain what determines the position of an intermediary's optimal mode in the spectrum spanning from a pure marketmaker mode to a pure middleman mode.

We consider a model in which the intermediated-market structure is determined endogenously as a result of the strategic choice of a monopolistic intermediary. In our model, there are two markets open to agents, one is an intermediated market operated by the intermediary, and the other is a decentralized market where buyers and sellers search individually. The intermediated market combines two business modes: as a middleman, the intermediary is prepared to serve many buyers at a time by holding inventories; as a marketmaker, the intermediary offers a platform and receives fees. The intermediary can choose how to allocate the attending buyers among these two business modes.

We formulate the intermediated market as a directed search market in order to feature the intermediary's technology of spreading price and capacity information efficiently – using the search function offered in the NYSE Arca or Expedia/Amazon website or in the web-based platform for house hunters. For example, one can receive instantly all relevant information such as prices, the terms of trade and stocks of individual sellers. In this setting, each individual seller is subject to an inventory capacity of discrete units (normalized to one unit in the model), whereas the middleman is subject to an inventory capacity of a mass K. Naturally, the middleman is more efficient in matching demands with supplies in a directed search equilibrium. The decentralized market represents an individual seller's outside option that determines the lower bound of his market utility.

With this set up, we consider two situations, *single-market search* versus *multiple-market search*. Under single market search, agents have to choose which market to search in advance, either the decentralized market or the intermediated market. This implies that the intermediary needs to subsidize buyers with their expected value in the decentralized market, but once they participate, the intermediated market operates without fear of competitive pressure outside. Given that the middleman mode is more efficient in realizing transactions, the intermediary uses the middleman-mode exclusively when agents search in a single market.

When agents are allowed to search in multiple markets, attracting buyers becomes less costly compared to the single-market search case — the intermediary does not need to subsidize buyers to induce participation. However, the prices/fees charged in the intermediated market must be acceptable relative to the available option in the decentralized market. Otherwise, buyers and sellers can easily switch to the outside market. Thus, under multiple-market search, the outside option creates competitive pressure to the overall intermediated market. In deciding the optimal intermediation mode, the intermediary takes into account that a higher middleman capacity induces more buyers to buy from the middleman, and fewer buyers to search on the platform. This has two opposing effects on its profits. On the one hand, a higher capacity of the middleman leads to more transactions in the intermediated market, and consequently to larger profits. On the other hand, sellers are less likely to trade on a smaller-scaled platform and buyers are more likely to trade with a larger scaled middleman, so that more sellers are available when a buyer attempts to search in the decentralized market. Accordingly, buyers expect a higher value from the less tight decentralized market. This causes cross-markets feedback that leads to competitive pressure on the price/fees that the intermediary can charge, and a downward pressure on its profits. Hence, the intermediary trade-offs a larger quantity against lower price/fees to operate as a larger-scaled middleman. This trade-off determines the middleman's selling capacity and eventually the intermediation mode.

Single-market search may correspond to the traditional search technology for local supermarkets or brick and mortar retailers. Over the course of a shopping trip, consumers usually have to search, buy and even transport the purchased products during a fixed amount of time. Given the time constraint, they visit a limited number of shops — typically one supermarket — and appreciate the proximity provided by its inventory. In contrast, multi-market search is related to the advanced search technologies that are available in the digital economy. It allows the online-customers to search and compare various options easily. Multiple market search is also relevant in the market for durable goods such as housing or expensive items where customers are exposed to the market for a sufficiently long time to ponder multiple available options.

We show that a marketmaking middleman can emerge in a directed search equilibrium. The marketmaking middleman can outperform either extreme intermediation mode. Relative to a pure market-maker, its inventory holdings can reduce the out-of-stock risk, while relative to a pure middleman its platform can better exploit the surplus of intermediated trade. It is this trade-off that answers the two puzzles above. Somewhat surprisingly, our result suggests that an improvement in search technologies induces the intermediary to generate inefficiencies to improve profits. This occurs via the use of the frictional platform that generates unmatched buyers who then search again but are also unmatched in the frictional decentralized market.

We offer various extensions to our baseline model. First, we introduce non-linear matching functions in the decentralized market, which increases the profitability of middleman even with multi-market search. Second, we introduce the aggregate resource constraint and frictions in the wholesale market, which increases the profitability of using an active platform even with single-market search. Third, we introduce a convex inventory-holding cost function, which reduces the profitability of a middleman, and sellers' purchase/production costs that accrue prior to entering the platform, which reduce the profitability of marketmaker. However, these extensions do not alter our main insight on the emergence of marketmaking middlemen. Forth, we introduce competing intermediaries. As is consistent with the monopoly analysis, we show that an active platform of an incumbent intermediary that charges positive fees can only be profitable when agents search in multi markets and the other intermediary enters with an active platform.

Finally, we examine empirically the implication of our theory. Just like in the last extension of competing intermediaries, we take Amazon as the centralized market and eBay as the decentralized market. For our chosen product category, Amazon acts as a marketmaking middleman: for 32% of the sample, Amazon acts as a middleman; for the other 68%, Amazon acts as a platform. Our empirical evidence strongly supports the model's prediction that Amazon is more likely to sell the product as a middleman when the chance of buyers to meet a seller in eBay is low, the buyers' bargaining power is low, and total demand is high.

This paper is related to the literature of middlemen developed by Rubinstein and Wolinsky (1987).[2] Using a directed search approach, Watanabe(2010, 2018a, 2018b) provides a model of an intermediated market operated by middlemen with high inventory holdings. The middlemen's high selling-capacity enables them to serve many buyers at a time, thus to lower the likelihood of stock-out, which generates a retail premium of inventories. This mechanism is adopted by the middleman in our model. Hence, if intermediation fees were not available, then our model would

[2] Rubinstein and Wolinsky (1987) show that an intermediated market can be active under frictions, when it is operated by middlemen who have an advantage in the meeting rate over the original suppliers. Given some exogenous meeting process, two main reasons have been considered for the middlemen's advantage in the rate of successful trades: a middleman may be able to guarantee the quality of goods (Biglaiser 1993, Li 1998), or to satisfy buyers' demand for a variety of goods (Shevchenko 2004). While these are clearly sound reasons for the success of middlemen, the buyers' search is modeled as an undirected random matching process, implying that the middlemen's capacity cannot influence buyers' search decisions in these models. See also Duffie et al. (2005), Lagos and Rocheteau (2009), Lagos et al. (2011), Weill (2007), Johri and Leach (2002), Masters (2007), Watanabe (2010), Watanabe (2018a), Wright and Wong (2014), Geromichalos and Jung (2018), Lagos and Zhang (2016), Awaya and Watanabe (2018b), Awaya and Watanabe (2018a), Nosal et al. (2015).

be a simplified version of Watanabe where we added an outside market. It is worth mentioning that in Watanabe(2010, 2018a, 2018b), the middleman's inventory is modeled as an indivisible unit, i.e., a positive integer, so that the middlemen face a non-degenerate distribution of their selling units as other sellers do. In contrast, here we model the inventory as a mass, assuming more flexible inventory technologies, so that the middleman faces a degenerate distribution of sales. This simplification allows us to characterize the middleman's profit-maximizing choice of inventory holdings — in Watanabe(2010, 2018b) the inventory level of middlemen is determined by aggregate demand-supply balancing, and in Watanabe (2018a) it is treated as an exogenous parameter. More recently, Holzner and Watanabe (2016) study a labor market equilibrium using a directed search approach to model a job-brokering service offered by Public Employment Agencies, but the choice of intermediation mode is not the scope of their paper.

Our paper is also related to the two-sided market literature.[3] The critical feature of a platform is the presence of a cross-group externality, i.e., the participants' expected gains from a platform depend positively on the number of participants on the other side of it. Caillaud and Jullien (2003) show that even when agents have a pessimistic belief on the intermediated market, the intermediary can make profits by using "divide-and-conquer" strategies, namely, subsidizing one group of participants in order to attract another group and extract the ensuing externality benefit. To be consistent with this literature, we develop an equilibrium with an intermediary based on similar pessimistic beliefs. Broadly speaking, if there were no middleman mode, our model would be a directed search version of Caillaud and Jullien (2003) in combination with a decentralized market. Further, our result that the intermediary sometimes induces agents to search more than they like is related to the idea of search diversion in Hagiu and Jullien (2011). They pursue this idea in a model of an information platform that has superior information about the match between consumers and stores and that could direct consumers first to their least preferred store.

Rust and Hall (2003) develop a search model which features the coexistence of different intermediation markets.[4] They consider two types of intermediaries, one is "middlemen" whose market requires costly search and the other is a monopolistic "market maker" who offers a frictionless mar-

[3]See, e.g. Rochet and Tirole (2003), Rochet and Tirole (2006), Caillaud and Jullien (2001), Caillaud and Jullien (2003), Rysman (2009), Armstrong (2006), Hagiu (2006), (Weyl, 2010). Related papers from other aspects can be found in Baye and Morgan (2001), Rust and Hall (2003), Parker and Van Alstyne (2005), Nocke et al. (2007), Galeotti and Moraga-González (2009), Loertscher and Niedermayer (2008), Edelman et al. (2015), Hagiu and Wright (2014), Condorelli et al. (2018), and Rhodes et al. (2017). Earlier contributions of this strand of literature are, e.g., Stahl (1988), Gehrig (1993), Yavaş (1994), Yavaş (1996), Spulber (1996), and Fingleton (1997). For platform studies emphasizing matching heterogeneity, see e.g., Bloch and Ryder (2000), Damiano and Li (2008) and De Fraja and Sákovics (2012).

[4]See Ju et al. (2010) who extend the Rust and Hall model by considering oligopolistic market makers.

ket. They show that agents segment into different markets depending on heterogeneous production costs and consumption values, thus these two types of intermediaries can coexist in equilibrium. Their model is very different from ours in many respects. For instance, selling capability and inventory do not play any role in their formulation of a search rule, but it is the key ingredient in our model. As Rust and Hall (2003) state: "An important function of intermediaries is to hold inventory to provide a buffer stock that offers their customers liquidity at times when there is an imbalance between supply and demand. In the securities business, liquidity means being able to buy or sell a reasonable quantity of shares on short notice. In the steel market, liquidity is also associated with a demand for immediacy so that a customer can be guaranteed of receiving shipment of an order within a few days of placement. *Lacking inventories and stockouts, this model cannot be used to analyze the important role of intermediaries in providing liquidity* (page 401; emphasis added)." This is exactly what we emphasize in our model which incorporates Rust and Hall's observation. We show that intermediaries can pursue different types of intermediation modes even when faced with homogeneous agents.

The rest of the paper is organized as follows. Section 2 presents our model of intermediation, and the benchmark case of single-market search. Section 3 extends the analysis to allow for multiple-market technologies and presents the key finding of our paper. Section 4 discusses modeling issues. Section 5 discusses some real-life applications of our theory. Section 6 presents the empirical evidence. Finally, section 7 concludes. Omitted proofs are in the Appendix. Finally, a Web Appendix contains our extension to allow for unobservable capacity and participation fees, and additional details on the empirical analysis.

2 A basic model with single-market search

This section studies the choice of intermediation mode for single-market technologies that serves as a benchmark of our economy. Along the way, we introduce the environment in which the monopolistic intermediary operates.

2.1 The framework

Agents We consider a large economy with two populations, a mass B of buyers and a mass S of sellers. Agents of each type are homogeneous. Each buyer has unit demand for a homogeneous good, and each seller is able to sell one unit of that good. The consumption value for buyers is normalized to 1. Sellers can purchase the good from a wholesale market. We assume there exists a competitive wholesale market, and the demand of all suppliers is always satisfied with a price equal to the marginal cost $c \in [0, 1)$.[5]

Retail markets Buyers and sellers can only meet each other in a retail market. There are two retail markets, a centralized market, which is operated by a monopolistic intermediary and a decentralized market. The decentralized market is the outside option for the buyers/sellers. Retail services can be exclusive or non-exclusive. Accordingly, we consider two different search technologies of buyers/sellers that correspond to both cases. This section spells out the details of single-market search where buyers/sellers can attend only one market, and Section 3 is about multi-market search where agents can attend both markets sequentially. We next describe price formation and the trading mechanisms in each market.

Matching and price formation in the decentralized market The decentralized market (hereafter D market) is featured by random matching and bilateral bargaining. Suppose all buyers and sellers participate in the D market, then a buyers meets a seller with probability λ^b and a seller meets a buyer with probability λ^s, satisfying $B\lambda^b = S\lambda^s, \lambda^b, \lambda^s \in (0, 1)$. If a subset of buyers $B^D \leq B$ and sellers $S^D \leq S$ participate, then the matching probabilities are $\lambda^b \times \frac{S^D}{S}$ and $\lambda^s \times \frac{B^D}{B}$, respectively.[6] This matching technology, which is linear in the participants on the other side of the market, is a simplified way to formulate the outside option of agents. In Section 5.1, we show that our main insight is valid with general non-linear matching functions where the meeting rate (and the expected value) depends on the relative measures of buyers and sellers. Matched partners follow an efficient bargaining process, which yields a linear sharing of the total surplus, with a share of $\beta \in (0, 1)$ for the buyer and a share of $1 - \beta$ for the seller.

Matching and price formation in the centralized market The centralized market (hereafter C market) is operated by a monopolistic intermediary whose profit-maximizing mode is the focus of the model. The intermediary can perform two different intermediation activities. As

[5] The reason that we do not explicitly model producers in the wholesale market is that they are passive anyway in our model. Without retail technologies producers are not able to serve buyers themselves but just supply the good in a competitive wholesale market.

[6] Imagine that all sellers have a slot at the D market, but not all are available. A seller is absent from the D market if he has sold the unit inventory in another market. If a buyer visits a seller who is out of stock, no transaction (or match) takes place. Given that a mass of $S^D < S$ sellers join the D market, the probability that a buyer can be successfully matched to an available seller is $\lambda^b \frac{S^D}{S}$. The same logic applies to λ^s. It is easy to verify that $B^D \lambda^b \frac{S^D}{S} = S^D \lambda^s \frac{B^D}{B}$.

a middleman, he purchases a good of mass $K \geq 0$ from the wholesale market at a cost c, and resells it to buyers at a price of $p^m \in [c, 1]$. As a market-maker, he does not buy and sell but instead provides a platform where buyers and sellers can interact with each other for trade after paying fees. Denote the fees in the market-maker sector by $\{f^b, f^s\}$, where $f^b,\ f^s \in [0, 1]$ is a transaction fee charged to a buyer or a seller, respectively, and satisfies $f^b + f^s \leq 1$. By choosing $\{K, p^m, f^b, f^s\}$, the intermediary fine-tunes the flows of buyers visiting the middleman and the market-maker sectors such that profits are maximized.[7]

Observing the intermediary's inventory K, price p^m, and fees $\{f^b, f^s\}$, buyers and sellers choose which market to participate in. Suppose that a mass of $B^C \in (0, B]$ buyers and a mass of $S^C \in [0, S]$ sellers have decided to participate in the C market, then trade in C market follows a directed search game, which consists of the following two stages.[8]

1. In the first stage, all participating sellers with a unit selling-capacity simultaneously post a price which they are willing to sell at. Owing to the advanced matching technology from the intermediary, the prices and capacities of all the individual suppliers (sellers and the middleman) are publicly observable within the C market.

2. Observing the price and capacity information, in the second stage, all buyers simultaneously decide which supplier to visit. As is standard in the literature, we assume that each buyer can visit at most one supplier, one of the sellers or the middleman.

Directed search equilibrium in the C market Since buyers cannot coordinate their actions over which supplier to visit, the C market is subject to coordination frictions. There is a chance that more buyers may show up at a given supplier than the supplier can accommodate, in which case some buyers get rationed (when multiple buyers show up, the seller picks one at random). Alternatively, fewer buyers may show up at a supplier than the supplier can accommodate, in which case the supplier is rationed. We consider a symmetric equilibrium where all individual sellers post the same price and all buyers use an identical mixed strategy to determine a seller to visit given the announced prices/fees. As is standard in the directed search literature, we do not consider equilibria in which buyers follow asymmetric strategies since this would require an unrealistic amount of implicit coordination. The expected queue lengths at the sellers adjust such

[7] Allowing for participation fees/subsidies, which accrue irrespective of transactions in the C market, will not affect our main result. In the Web Appendix, we offer such an extended model.

[8] The intermediary's announcement and the mass of participating agents are common knowledge.

that all buyers receive their market utility (maximum expected payoff that the buyer can obtain) *on* and *off* the equilibrium path. This implies that the lower the price that a seller posts, the longer the expected queue of buyers will be. In equilibrium, each individual seller maximizes profits by posting a price p^s (that implies an expected queue x^s of buyers) given the constraint that buyers will not visit if they receive less than their market utility. In a symmetric equilibrium, buyers visit each identical seller with equal probability.

Matching with the middleman Since the middleman has capacity K, the expected profit from the middleman sector is given by $\min\{K, x^m\}p^m$. The expected value for a buyer who visits the middleman is given by

$$V^m(x) = \min\{\frac{K}{x^m}, 1\}(1 - p^m),$$

where $\min\{\frac{K}{x^m}, 1\}$ is the matching probability of a buyer at the middleman.

Matching with individual sellers on the platform The matching probability at each individual seller follows an urn-ball matching function. The number of buyers visiting an individual seller is a random variable, denoted by N, which follows a Poisson distribution, $\text{Prob}[N = n] = \frac{e^{-x}x^n}{n!}$, with an expected queue of buyers $x \geq 0$.[9] With limited selling capacity, each seller is able to serve only one buyer. A seller with an expected queue $x^s \geq 0$ has a probability $1-e^{-x^s}$ ($= \text{Prob}[N \geq 1]$) of successfully selling, while each buyer has a probability $\eta^s(x^s) = \frac{1-e^{-x^s}}{x^s}$ of successfully buying. Hence, the expected value of a seller in the platform with a price p^s and an expected queue x^s is given by

$$W(x^s) = x^s \eta^s(x^s)(p^s - f^s - c),$$

while the expected value of a buyer who visits the seller is

$$V^s(x^s) = \eta^s(x^s)(1 - p^s - f^b).$$

In equilibrium, the queues x^s and x^m should satisfy two requirements. The first requirement is a standard accounting identity,

$$S^C x^s + x^m = B^C, \tag{1}$$

[9] This is due to coordination frictions. Suppose there are b buyers and s sellers. If each buyer visit each seller with equal probability, any seller gets a buyer with probability $1 - (1 - \frac{1}{s})^b$. Taking the limit as b and s go to infinity and $x^s = b/s$ fixed, in a large market as we propose here, a fraction $1 - e^{-x^s}$ of sellers get a buyer. This process is called an urn-ball matching function. See the seminal work by Peters (1991, 2000).

which states that the number of buyers visiting individual sellers $S^C x^s$ and the middleman x^m should sum up to the total population of participating buyers B^C. The second requirement is that buyers search optimally and should visit only sellers who offer them their market utility, implying that

$$x^m = \begin{cases} B^C & \text{if } V^m(B^C) \geq V^s(0) \\ (0, B^C) & \text{if } V^m(x^m) = V^s(x^s) \\ 0 & \text{if } V^m(0) \leq V^s(\frac{B^C}{S^C}), \end{cases} \tag{2}$$

where $V^i(x^i)$ is the equilibrium value of buyers in the C market of visiting a seller if $i = s$ and the middleman if $i = m$. Note, the third case of (2) happens only if $S^C > 0$. Combining (1) and (2) gives the counterpart for $x^s \in [0, \frac{B^C}{S^C}]$. Based on the mass of buyers visiting middleman and marketmaker, we define the intermediation mode.

Definition 1 (Intermediation Mode) *Suppose $B^C \in (0, B]$ buyers and $S^C \in [0, S]$ sellers participate in C market. Then, given the equilibrium search conditions (1) and (2), we say that the intermediary acts as:*

- *a pure middleman if $x^m = B^C$;*
- *a market-making middleman if $x^m \in (0, B^C)$;*
- *a pure market-maker if $x^m = 0$.*

Pessimistic beliefs and participation in the C market Below, we describe the participation rule that determines B^C and S^C under single-market search. As in Caillaud and Jullien (2003), we assume agents hold pessimistic beliefs on the participation decision of agents on the other side of the C market. Under such belifs, buyers and sellers coordinate on a participation rule such that the C market is empty whenever possible. In the framework of Caillaud and Jullien (2003), a divide-and-conquer strategy is the only way to get a positive market share. In our model, however, even without the participation of sellers, there is supply in the C market — the inventory K of the middleman is always there. Therefore, to break the pessmistic beliefs, the intermediary must convince buyers that if they do join the C market, the expected value is higher than if they join

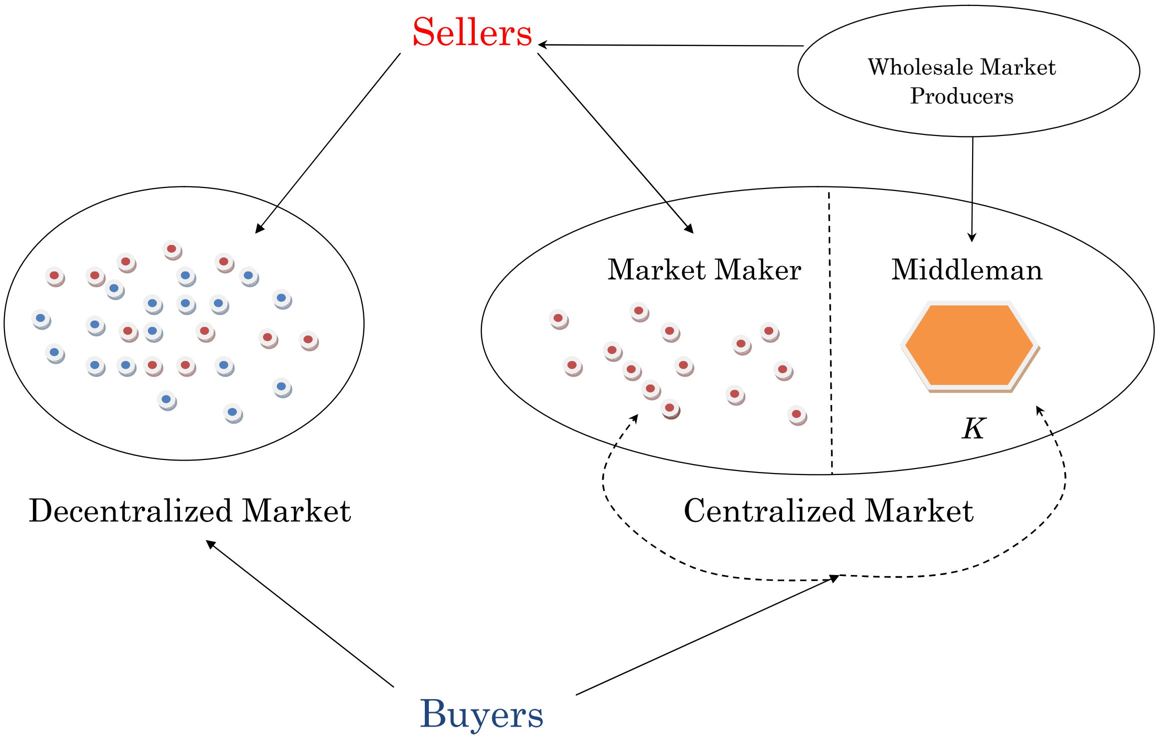

Figure 1: Overview

Note: The figure depicts the model abstracting from the timing and the price/fee structure. Buyers are represented by blue dots. Sellers are represented by red dots. They can participate in the D market where they are matched following a linear matching technology. Alternatively, they can participate in the C market which has two sectors: A market-maker sector which consists of individual sellers and a middleman sector which is supplied by the intermediary. A buyer can choose one of the sellers or the middleman to visit taking into account both the price and matching probability.

the D market. That is,[10]

$$\max\{V^m(x^m), V^s(x^s)\} \geq \lambda^b \beta(1-c). \tag{3}$$

Whenever condition (3) holds, all buyers join the C market $B^C = B$.

Timing We summarize the set-up of the model by the timing structure.

1. Two retail markets, a C market and a D market, open. In the C market, the intermediary decides whether or not to activate the middleman sector and/or the platform. The intermediary announces middleman capacity K and price p^m, and a set of fees $\{f^b, f^s\}$ in the

[10] Under such a belief, Caillaud and Jullien (2003) propose that the intermediary can charge negative fees (give subsidy) to induce the participation of agents. In our setup, the middleman's capacity advantage reveals that supply K is available in the C market, irrespective of the number of sellers participating. Hence, the intermediary can induce buyers' participation under those beliefs, as long as condition (3) is satisfied. When the middleman's supply is not observable in the participation stage, we will offer a version of our model with participation fees/subsidies in the Web Appendix, and show that our main result is still valid.

platform sector.

2. Observing the announced capacity, price and fees, buyers and sellers simultaneously decide whether to participate in the C market under pessimistic belifs.

3. In the C market, the participating buyers, sellers and middleman are engaged in a directed search game. In the D market, agents search randomly and follow the efficient sharing rule for the trade surplus.

Figure 1 gives an overview of our model. Sellers are represented by red dots and buyers are represented by blue dots. They may engage in random matching in the Decentralized Market or, alternatively, they may meet in the Centralized Market. The market-maker sector of the C market consists of sellers with one unit each. The intermediary can also open a middleman sector with a continuum of inventory. Buyers who visit the C market can choose which individual supplier to visit or to visit the middleman and are subject to coordination frictions.

Given the characterization of the directed search equilibrium in the C market and the random matching in the D market, working backward, we turn to the profit-maximizing problem of the intermediary below.

2.2 Optimal intermediation mode under single-market search

Given the directed search equilibrium, the intermediary chooses inventory capacity K, price/fees p^m, f^b, f^s to tune his business mode, and ultimately maximize his profits. In what follows, we show that if agents have a single-market search technology, then the intermediary will not open the platform, inducing $S^C = 0$, and will serve all buyers $B^C = B$ as a pure middleman with $K = B$ and $x^m = B$. This leads to the following pure middleman profits,

$$\Pi = B(p^m - c),$$

subject to the participation constraint of buyers in the C market,

$$V^m(x^m) = 1 - p^m \geq \lambda^b \beta(1 - c). \tag{4}$$

The middleman sets $p^m = 1 - \lambda^b \beta(1 - c)$. Note here that the outside value of buyers is given by $\lambda^b \beta(1 - c)$, which is supported by their belief that the D market is non-empty. We make the usual tie-breaking assumption that agents choose to participate in a market if they are indifferent.

Now, we show that creating an active platform is not profitable. Suppose that the intermediary opens a platform with $S^C = S$ sellers and intermediation fees $f = f^b + f^s \leq 1$. Then, the platform generates a non-negative trade surplus $1 - f \geq 0$. In any active platform under pessimistic beliefs, it must be that $V^s(x^s) \geq \lambda^b\beta(1-c)$ and $p^s \geq f^s + c$. These conditions imply $f = f^s + f^b < (1-\lambda^b\beta)(1-c)$. The requirement for an active middleman is that the inventory price $p \leq 1 - \lambda^b\beta(1-c)$. These are the constraints confronted by the intermediary to break pessimistic beliefs. Then, the intermediary's expected profits consist of the revenue of platform fees, $S(1-e^{-x^s})f$, and the revenue of inventory sales minus inventory cost, $\min\{K,x\}p - Kc$. Without going into the optimization problem, observe that

$$\begin{aligned}
\Pi(x,f,p,K) &= S(1-e^{-x^s})f + \min\{K,x\}p - Kc \\
&< Sx^s f + xp - Kc \\
&\leq (Sx^s + x)\max\{f,p\} - Kc \\
&\leq B(1-\lambda^b\beta)(1-c) = \Pi,
\end{aligned}$$

for all $x^s \in (0, \frac{B}{S}]$. Hence, opening the platform is not profitable.

The intuition behind the occurrence of a pure middleman mode is as follows. Given the frictions on the platform, a larger middleman sector creates more transactions. To achieve the highest possible number of transactions, the intermediary shuts down the platform. In a nutshell, the middleman's capacity is the most efficient way to distribute the good and, if agents search within a single market, the intermediary is guaranteed the highest possible surplus by choosing this mode The allocation characterized here serves as a benchmark for the rest of our analysis.

Proposition 1 (Pure middleman) *Given single-market search technologies, the intermediary will not open the platform and will act as a pure middleman with $x^m = K = B$, serving all buyers for sure.*

3 Multi-market search

In this section, we extend our analysis to multiple-market search technologies where agents can search in both the C and the D market.

Opening sequence To facilitate the presentation of our key idea, we assume that the C market opens prior to the D market.[11] Apart from the fact that this appears to be the most natural setup in our economy, it can be motivated by the first mover advantage of the intermediary: its expected profit is higher if the C market opens before the D market. Hence, this sequence arises endogenously if the intermediary is allowed to select the timing of the market sequence.[12]

Participation in the C market Under multi-market search, participating in the C market does not rule out the possibility of trading in the D market. Hence, for the intermediary, convincing agents to participate is not difficult. Indeed, the opening sequence makes it natural for all agents to first visit the C market and then the D market. The more difficult part for the intermediary is to convince agents that trade in the C market is better than continuing to search in the D market. The complications come from the fact that the terms of trade that the monopolist commits to in the C market affect the market utility in of buyers and sellers in the D market. The competition from the D market is reflected in three participation constraints on prices/fees that we will discuss below: (5), (6) and (7) [13]

3.1 Directed search equilibrium under multi-market search

We work backward and first characterize the directed search equilibrium for the C market, given that the market-maker sector is active and that such an equilibrium exists. The intermediary ex ante commits to the following tuple $\{K, p^m, f^b, f^s\}$ which is observed by the buyers.

Participation constraints to trade in the C market As under single-market search, any directed search equilibrium in the C market has to satisfy (1) and (2). Given the multiple-market search technology, what is new here is that agents expect a non-negative value of visiting the D market when deciding whether or not to accept an offer in the C market. Whenever the platform

[11] If the two markets opened at the same time, we would have to deal with the agents' belief about what other agents would choose when they turn out to be matched in both markets. This would give rise to the multiplicity of equilibria which complicates the analysis significantly. Our sequential setup avoids this issue. In an infinite horizon model, one can construct a stationary equilibrium relatively easily where the order of the markets does not matter (see Watanabe 2018a).

[12] In a recent study without intermediation, Armstrong and Zhou (2015) show that a seller often makes it harder or more expensive to buy its product later than at the first opportunity.

[13] Note further that irrespective of agents' belief, an empty D market cannot occur in equilibrium. This is because even when buyers are extremely pessimistic about the D market so that sellers are indifferent between entering and not entering, there will always be sellers who fail to sell in the C market and they will be automatically present in the D market.

is active, that is $x^s > 0$ and $S^C = S$, it must satisfy the following participation constraints:

$$1 - p^s - f^b \geq \lambda^b e^{-x^s} \beta (1 - c), \tag{5}$$

$$p^s - f^s - c \geq \lambda^s \xi(x^m, K)(1 - \beta)(1 - c). \tag{6}$$

The participation constraint for buyers to trade in the C market,(5) states that the offered price/fee in the platform is acceptable only if the offered payoff, $1 - p^s - f^b$, weakly exceeds the expected value that buyers can obtain in the D market: the outside payoff is $\beta(1-c)$ if the buyer matches with a seller who has failed to trade in the C market. This happens with probability $\lambda^b e^{-x^s}$. Hence, the larger the platform size x^s, the higher the chance that a seller trades in the C market, and the lower the chance that a buyer can trade successfully in the D market and the lower his expected outside payoff.

The participation constraint for sellers to trade in the C market(6) states that the payoff in the C market $p^s - f^s - c$ should be no less than the expected payoff in the D market. This payoff depends on a seller's chance of finding a trading partner in the D market $\lambda^s \xi(x^m, K)$, where $\xi(x^m, K)$ represents the probability that a buyer has failed to trade in the C market and is given by

$$\xi(x^m, K) \equiv 1 - \frac{1}{B}\left(\min\{K, x^m\} + S\left(1 - e^{-\frac{B - x^m}{S}}\right)\right).$$

The buyer visits the middleman sector with probability $\frac{x^m}{B}$ and is served with probability $\min\left\{\frac{K}{x^m}, 1\right\}$, or he visits the platform with probability $\frac{Sx^s}{B}$ and is served with probability $\eta^s(x^s) = \frac{1-e^{-x^s}}{x^s}$. Hence, in the above expression, the second term represents the expected chance of the buyer to trade in the C market. A similar participation constraint must be satisfied in order for buyers to visit the middleman sector:

$$1 - p^m \geq \lambda^b e^{-x^s} \beta (1 - c), \tag{7}$$

where the middleman's price must be acceptable for buyers relative to the expected payoff in the D market.

Note that under these participation constraints, all buyers/sellers are weakly better off trading in the C market. Hence, under the tie-breaking assumption that agents choose the C market when indifferent, all matched pairs trade at C when the participation constraints are satisfied. That is, $B^C = B, S^C = S$. However, the tie-breaking assumption is not crucial. This is because the intermediary can lower $f^i, i = b, s$ slightly so that (5) to (7) hold strictly, and $B^C = B, S^C = S$.

In the above formulation, all agents are always present in the D market irrespective of whether they trade in the C market or not. This implies that a buyer can fail to transact in the D market because he either met a seller in the D market who already sold its item in the C market or because he could not find a seller. This creates an interaction between the C market and the D market.

Equilibrium values Given the outside option that buyers can obtain in the D market, the equilibrium value of buyers in the C market equals $V = \max\{V(x^s), V(x^m)\}$, where

$$V^s(x^s) = \eta^s(x^s)\left(1 - p^s - f^b\right) + \left(1 - \eta^s(x^s)\right)\lambda^b e^{-x^s}\beta(1-c) \tag{8}$$

for an active platform $x^s > 0$ and

$$V^m(x^m) = \min\{\frac{K}{x^m}, 1\}(1 - p^m) + \left(1 - \min\{\frac{K}{x^m}, 1\}\right)\lambda^b e^{-x^s}\beta(1-c) \tag{9}$$

for an active middleman sector $x^m > 0$. In this case, if a buyer visits a seller (or a middleman), then he gets served with probability $\eta^s(x^s)$ (or $\eta^m(x^m)$) and his payoff is $1 - p^s - f^b$ (or $1 - p^m$). If not served in the C market, he enters the D market and finds an available seller with probability $\lambda^b e^{-x^s}$. Similarly, the equilibrium value of active sellers in the platform is given by

$$W(x^s) = x^s\eta^s(x^s)(p^s - f^s - c) + \left(1 - x^s\eta^s(x^s)\right)\lambda^s\xi(x^m, K)(1-\beta)(1-c). \tag{10}$$

A seller trades successfully in the C market platform with probability $x^s\eta^s(x^s)$ and if this occurs, he receives $p^s - f^s - c$. If not successful in the C market, the seller can meet a buyer in the the D market with probability $\lambda^s\xi(x^m, K)$ and obtains a payoff of $(1-\beta)(1-c)$.

We now need to determine the equilibrium price p^s. We follow the standard procedure in the directed search literature. Suppose a seller deviates to a price $p \neq p^s$ that attracts an expected queue $x \neq x^s$ of buyers. Note that given the limited selling-capacity, this deviation has measure zero and does not affect the expected utility in the C market, V. Since buyers must be indifferent between visiting any seller (including the deviating seller), the equilibrium market-utility condition holds on and off the equilibrium path and satisfies,

$$V = \eta^s(x)\left(1 - p - f^b\right) + \left(1 - \eta^s(x)\right)\lambda^b e^{-x^s}\beta(1-c), \tag{11}$$

where $\eta^s(x) \equiv \frac{1-e^{-x}}{x}$ is the probability that a buyer is served by this deviating seller. If not served, which occurs with probability $1 - \eta^s(x)$, his expected utility in the D market is $\lambda^b e^{-x^s}\beta(1-c)$.

Given market utility V, (11) determines the relationship between x and p, which we denote by $x = x(p|V)$. This standard directed search logic yields a downward sloping demand curve faced by the seller: when the seller raises his price p, the queue length of buyers x becomes smaller and this corresponds to a lower trading probability, and vice versa.

Given the search behavior of buyers described above and the market utility V, the seller's optimal price must satisfy

$$p^s(V) = \arg\max_p \left\{ \begin{array}{c} \left(1 - e^{-x(p|V)}\right)(p - f^s - c) \\ +e^{-x(p|V)} \lambda^s \xi(x^m, K)(1-\beta)(1-c) \end{array} \right\}.$$

Substituting out p using (11), the sellers' objective function can be written as

$$W(x) = \left(1 - e^{-x}\right)(v(x^m, K) - f) - x\left(V - \lambda^b e^{-x^s} \beta(1-c)\right) + \lambda^s \xi(x^m, K)(1-\beta)(1-c),$$

where $x = x(p|V)$ satisfies (11) and

$$v(x^m, K) \equiv \left[1 - \lambda^b e^{-\frac{B - x^m}{S}} \beta - \lambda^s \xi(x^m, K)(1-\beta)\right](1-c)$$

is the intermediated trade surplus, i.e., the total trading surplus in the C market net of the outside options. Since choosing a price is isomorphic to choosing a queue, the first order condition is

$$\frac{\partial W(x)}{\partial x} = e^{-x}(v(x^m, K) - f) - \left(V - \lambda^b e^{-x^s} \beta(1-c)\right) = 0.$$

The second order condition can be easily verified. Arranging the first order condition using (11) and evaluating it at $x^s = x(p^s|V)$, we obtain the equilibrium price $p^s = p^s(V)$ which can be written as

$$p^s - f^s - c = \left(1 - \frac{x^s e^{-x^s}}{1 - e^{-x^s}}\right)(v(x^m, K) - f) + \lambda^s \xi(x^m, K)(1-\beta)(1-c). \tag{12}$$

Participatipon constraints revisited We can now rewrite the participation constraints (5) and (6) by substituting in (12). This yields

$$f \leq v(x^m, K), \tag{13}$$

which states that for the platform to be active $x^s > 0$, the total transaction fee f should not be greater than the intermediated trade surplus, $v(x^m, K)$. Whenever (5) and (6) are satisfied, (13) must hold, and whenever (13) is satisfied, (5) and (6) must hold. Hence, (13) is a sufficient condition for an active platform.

Observe that for $K < x^m$, we have

$$v(x^m, K) = \left[1 - \lambda^b e^{-\frac{B-x^m}{S}}\beta - \lambda^s\left(1 - \frac{K + S(1 - e^{-\frac{B-x^m}{S}})}{B}\right)(1-\beta)\right](1-c),$$

which is decreasing in x^m. This occurs because a larger sized platform (i.e., a lower x^m) crowds out the D market transactions and lowers the outside option of the buyers.

3.2 Intermediation mode

Our next step is to determine the profit for each intermediation mode, denoted by $\tilde{\Pi}(x^m)$.

Pure middleman: If the intermediary does not open the platform then $x^m = B$ and any encountered seller in the D market is always available for trade. Hence, as before, the middleman selects capacity $K = B$, serves all buyers at a price $p^m = 1 - \lambda^b\beta(1-c)$, satisfying (7), and unit cost (or wholesale price) c, and makes profits

$$\tilde{\Pi}(B) = B(1 - \lambda^b\beta)(1-c). \tag{14}$$

Pure market-maker: When the middleman sector is not open, $x^s = \frac{B}{S}$. Given that the equilibrium price p^s at the platform is given by (12), the intermediary charges a fee $f = f^b + f^s$ in order to maximize

$$S\left(1 - e^{-\frac{B}{S}}\right)f,$$

subject to the constraint (13). The constraint is binding and this yields:

$$f = v(0,0) = \left[1 - \lambda^b e^{-x^s}\beta - \lambda^s\xi(0,0)(1-\beta)\right](1-c).$$

where $\xi(0,0) = 1 - \eta^s(x^s)$. The profit for the market-maker mode is

$$\tilde{\Pi}(0) = S(1 - e^{-\frac{B}{S}})v(0,0). \tag{15}$$

Market-making middleman: If the intermediary is a market-making middleman, then $x^m \in (0, B)$ and $x^s \in \left(0, \frac{B}{S}\right)$, satisfying $V^m(x^m) = V^s(x^s)$. Using the equilibrium values: (8), (9), and (12), this indifference condition generates the following expression for the price $p^m = p^m(x^m)$:

$$p^m = 1 - \lambda^b e^{-x^s}\beta(1-c) - \frac{x^m e^{-x^s}}{\min\{K, x^m\}}(v(x^m, K) - f). \tag{16}$$

Together with (1), this equation defines the relationship between p^m and x^m. Applying this expression, we can see that the condition (7) is eventually reduced to (13). The profit for the marketmaking middleman mode is

$$\tilde{\Pi}(x^m) = \max_{x^m, f, K} \Pi(x^m, f, K) = S(1 - e^{-x^s})f + \min\{K, x^m\} p^m - Kc$$

subject to (13) and $x^m \in (0, B)$. Note that $K > x^m$ cannot be profitable since it is a mere increase in capacity costs. Profit maximization requires the following.

Lemma 1 *The market-making middleman sets: $K = x^m$ and $f = v(x^m, K)$.*

Proof. See the Appendix. ■

The above conditions imply that the intermediary's capacity should satisfy all the forthcoming demands, and the intermediation fee should be set to extract the full intermediation surplus.

Profit-maximizing intermediation mode: We are now in the position to derive the profit-maximizing intermediation mode. To do so, it is important to observe that relative to the pure middleman mode, an active platform with multiple-market search can undermine the D market by lowering the available supply. This influences the middleman's price in the following way. With $v(\cdot) = f$, the incentive constraint (7) is binding, and the middleman's equilibrium price is given by

$$p^m = 1 - \lambda^b e^{-x^s} \beta(1 - c)$$

for any $x^s \geq 0$ (see (16)). This shows that p^m decreases with x^m: the outside option of buyers depends positively on the size of the middleman sector, since a larger scale of the middleman crowds out the platform and increases the chance that a buyer can find an active seller in the D market (who was unsuccessful at the platform). Hence, in order to extend the size of the middleman sector, the intermediary must lower the price p^m. In other words, a larger platform allows for a price increase by reducing agents' outside trade opportunities.

Proposition 2 (Market-making middleman/Pure Market-maker) *Given multi-market search technologies, there exists a unique directed search equilibrium with active intermediation. The intermediary will open a platform and act as:*

- *a market-making middleman if* $\lambda^b\beta \leq \frac{1}{2}$ *or if* $\lambda^b\beta > \frac{1}{2}$ *and* $\frac{B}{S} \geq \bar{x} \in (0, \infty)$*;*

- *a pure market-maker if* $\lambda^b\beta > \frac{1}{2}$ *and* $\frac{B}{S} < \bar{x}$*.*

Proof. See the Appendix. ■

With multiple-market search technologies, there is a cross-market feedback from the D market to the C market, which makes using the platform as part or all of its intermediation activities profitable. Additionally, the intermediary must decide whether or ot it wants to operate as a pure market maker. Our result shows that the equilibrium mode of the intermediary depends on parameter values. If $\lambda^b\beta \leq \frac{1}{2}$ then the buyers' outside option is low. In this case, the middleman sector generates high enough profits for the market-making middleman mode to be adopted for any value of $\frac{B}{S}$. If instead $\lambda^b\beta > \frac{1}{2}$ then the buyers' outside option is high, and attracting buyers to the middleman sector is costly. In this case, the intermediary will act as a market-making middleman if $\frac{B}{S}$ is high, where the D market is tight for buyers and they expect a low value from it, and as a pure market maker if $\frac{B}{S}$ is low, where buyers expect a high value from the D market. Indeed, the same logic applies to the following comparative statics result.

Corollary 1 (Comparative statics) *Consider a parameter space in which the market-making middleman mode is profit-maximizing. Then, an increase in buyer's bargaining power* β *or buyer's meeting rate* λ^b *in the D market, or a decrease in the buyer-seller population ratio,* $\frac{B}{S}$*, leads to a smaller middleman sector* x^m *and a larger platform* x^s*.*

Proof. See the Appendix. ■

4 Extensions

This section considers extensions of the model. As we show below, our main insight, that the profit of using a platform as part or all of the intermediation business is relatively large when agents can search in multiple markets rather than in a single market only, is robust to these extensions.[14]

4.1 Matching functions

So far, we assumed a linear matching function in the D market. In this section, we allow for a more general matching function. As is standard in the literature we assume that the matching function

[14]For expositional simplicity, we let $c = 0$ and make the tie-breaking assumption that when the middleman is indifferent between $K = x^m$ and $K > x^m$ we set $K = x^m$.

is homogeneous of degree one in B^D and S^D, $M(1, \frac{1}{x^D}) = \frac{M(B^D, S^D)}{B^D}$ and $M(x^D, 1) = \frac{M(B^D, S^D)}{S^D}$, where $x^D = \frac{B^D}{S^D}$ is the buyer-seller ratio in the D market. Then, we allow for the dependence of the individual match probabilities on the buyer-seller ratio.

$$\lambda^b(x^D) = M(1, \frac{1}{x^D}) \text{ and } \lambda^s(x^D) = M(x^D, 1) = x^D \lambda^b(x^D) \tag{17}$$

where $\lambda^b(x^D)$ is strictly concave and decreasing in x^D.

With single-market search technologies, the result will not be affected by this extension (for instance, if we use the pessimistic belief assumption, then the matching probability in the D market is simply replaced by another constant $\lambda^i(x^D)$, $i = b, s$, with $x^D = \frac{B}{S}$). Therefore, we only consider multi-market search technologies. As mentioned before, we let agents exit if they have traded successfully in the C market, because if agents stayed in the D market as in the previous section, then again the analysis would remain essentially unchanged. Then, the population in the D market is given by

$$B^D = B - \min\{x^m, K\} - S(1 - e^{-x^s}) \text{ and } S^D = Se^{-x^s}.$$

With this modification, the buyers' probability to meet an available seller changes from $\lambda^b e^{-x^s}$ to $\lambda^b(x^D)$, and the sellers' probability to meet an available buyer changes from $\lambda^s \xi(x^m, K)$ to $\lambda^s(x^D) = x^D \lambda^b(x^D)$.

In what follows, we derive a condition for a pure middleman mode to be selected under multi-market search technologies. This is the case when, for example, $\lambda^{b\prime}(x^D) = 0$, i.e., when there is no feedback from the D-market to the intermediary's decision in the C market. We proceed with the following steps. First, note that, as before, there is no gain from having excess capacity $K > x^m$. In addition, a pure middleman wants to avoid stockouts $K < x^m$ if

$$\frac{d\tilde{\Pi}(K)}{dK} = \frac{d}{dK} K \left(1 - \lambda^b(x^D)\beta\right) = 1 - \lambda^b(x^D)\beta + \frac{K}{S}\lambda^{b\prime}(x^D)\beta > 0,$$

for any $x^D = \frac{B-K}{S} \geq 0$, which states that the elasticity of the middleman's price $p^m = 1 - \lambda^b(x^D)\beta$ should satisfy

$$z(K) \equiv -\frac{\partial p^m / \partial K}{p^m / K} = -\frac{K\lambda^{b\prime}(x^D)\beta}{S(1 - \lambda^b(x^D)\beta)} \leq 1.$$

This condition guarantees that a pure middleman should satisfy all the forthcoming demand $K = x^m$.

Second, when all buyers are served by the middleman $x^m = K = B$, the marginal gain of allocating buyers to the platform, measured by the intermediation fee,

$$f = 1 - \lambda^b(x^D)\beta - x^D\lambda^b(x^D)(1-\beta),$$

can not exceed the marginal opportunity cost, measured by the lost revenue in the middleman sector,

$$1 - \lambda^b(0)\beta - K\lambda^{b\prime}(0)\beta\frac{dx^D(K, x^s(K))}{dK}\,|_{x^s(K)=0},$$

where $x^s(K) = \frac{B-K}{S}$ and

$$\frac{dx^D(K, x^s(K))}{dK}\,|_{x^s(K)=0} = \frac{d}{dK}\frac{B-K-S(1-e^{-x^s(K)})}{Se^{-x^s(K)}}\,|_{x^s(K)=0} = \frac{-S+(B-K-S)}{S^2e^{-x^s(K)}}\,|_{K=B} = 0.$$

Hence, the intermediary can be a pure middleman even with multiple-market search technologies.

Proposition 3 *With a non-linear matching function in the D market outlined above, a pure middleman mode can be profitable even with multi-market search technologies if the middleman's price is inelastic at full capacity $x^m = K = B$. Otherwise, the intermediary should be a marketmaking middleman or a pure market maker.*

Proof. See the Appendix. ■

Figure 2 plots the size of the middleman sector $\frac{x^m}{B}$ and the elasticity of the middleman's price with respect to capacity, evaluated at $x^m = K = B$.[15] It shows that when a pure middleman mode is selected $\frac{x^m}{B} = 1$ the price is inelastic: $z(B) < 1$, whereas when an active platform is used the price is elastic: $z(B) > 1$. This confirms that given the appropriate restriction on the meeting rate $\lambda^b(x^D)$, our main conclusion in the baseline model is valid with an alternative assumption that agents exit after successful trade in the C market. It is intuitive that when the middleman's price is elastic, there is strong enough negative feedback from the D market on the price that makes the exclusive use of the middleman sector not profitable.

4.2 Endowment economy

In our baseline model, we simplified the middleman's inventory stocking by assuming that the good is supplied by competitive producers in the frictionless wholesale market. In this section, we study

[15] The figures is drawn with $S = 1$ and $\lambda^b(x^D) = \frac{1-e^{-x^D}}{x^D}$.

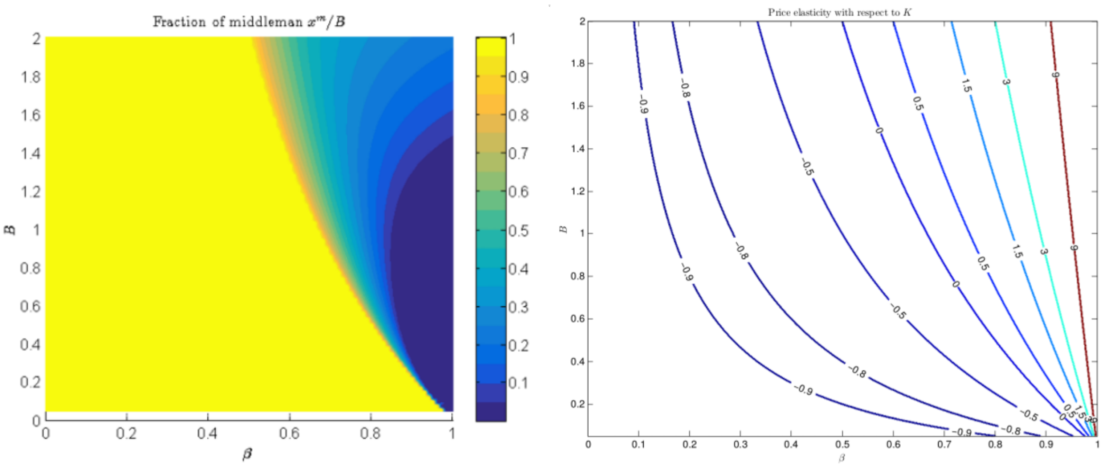

Figure 2: Size of middleman sector $\frac{x^m}{B}$ (Left) and Price elasticity $z(B)$ (Right) with Non-linear matching function

the implication of wholesale-market frictions in an endowment economy. Suppose that each seller owns one unit of endowment. In total, a mass of S commodities are available. In the wholesale market, the middleman can access a fraction α of sellers, where we assume that $\alpha \in (0, 1)$ is an exogenous parameter. Then, the middleman's inventory should satisfy the aggregate resource constraint,

$$K \leq \alpha S. \tag{18}$$

In a world with unlimited production capacity, sellers are willing to supply as long as the wholesale price, denoted by p^w, is enough to compensate for the marginal cost; whereas in an endowment economy, sellers are only willing to supply if p^w is high enough to compensate for trading opportunities they lose in other channels. Once contacted by the middleman, sellers choose among selling the endowment to the middleman, or joining the C market platform and/or joining the D market. To simplify the analysis, we abstract from the influence of what sellers can expect from the D market on the determination of the wholesale price, and assume that sellers in the D market receive zero trade share,

$$\beta = 1.$$

Our main conclusion does not depend on this simplification. Then, the middleman's offer to buy from sellers is accepted if and only if

$$p^w \geq W(x^s), \tag{19}$$

where $W(x^s)$ is the expected value of sellers to operate in the C market platform.

Single-market search: The determination of the intermediation mode depends on the available resources. If $B \leq \alpha S$, then the middleman can stock the full inventory to cover the entire population of buyers. In this case, by closing the platform $S^C = 0$, the middleman makes the highest possible profit, $\Pi = B(1-\lambda^b)$, with the wholesale price $p^w = 0$, just like in the baseline model. If $B > \alpha S$, then the middleman's inventory will not be enough to cover all buyers, and so the intermediary may wish to use a platform even under a single-market search technology. With the wholesale price p^w determined by the binding constraint, (19), the fee f and the price p^m determined by the binding participation constraint of buyers, $V = \max\{V^s(x^s), V^m(x^m)\} = \lambda^b$, the intermediary's problem can be written as the choice of the size of its inventory K and the allocation x^m that maximizes

$$\Pi\left(x^m, f, K\right) = (S-K)(1-e^{-x^s})f + \min\left\{K, x^m\right\}p^m - Kp^w$$

where $x^s = \frac{B-x^m}{S-K}$, subject to the resource constraint (18). To guarantee non-negative price/fees/profits, we shall assume sufficiently low values of $\lambda^b > 0$ whenever necessary (see the proof of Proposition 4).

As expected, the solution is characterized by the binding resource constraint (18) and an active platform $x^s > 0$ when $B > \alpha S$. Note that the intermediary could deactivate the platform since it would lead to the lowest wholesale price of middleman $p^w = 0$. However, it turns out that the benefit of fee revenue from the active platform outweighs the cost savings in the middleman sector. Hence, even with single-market technologies, the aggregate resource constraint can be one reason for the intermediary to open the platform sector in the endowment economy.

Proposition 4 *Consider the endowment economy outlined above with single-market search technology, and the zero trade share of sellers in the D market. The intermediation chooses to be:*

- *a pure middleman if* $B \leq \alpha S$;
- *a market-making middleman with* $K = \alpha S \leq x^m$ *if* $B > \alpha S$.

Proof. See the Appendix. ■

The result $x^m \geq K$ occurs because, in line with the previous setup, an excess inventory means extra costs in the middleman sector and lost revenues in the platform. Figure 3 demonstrates

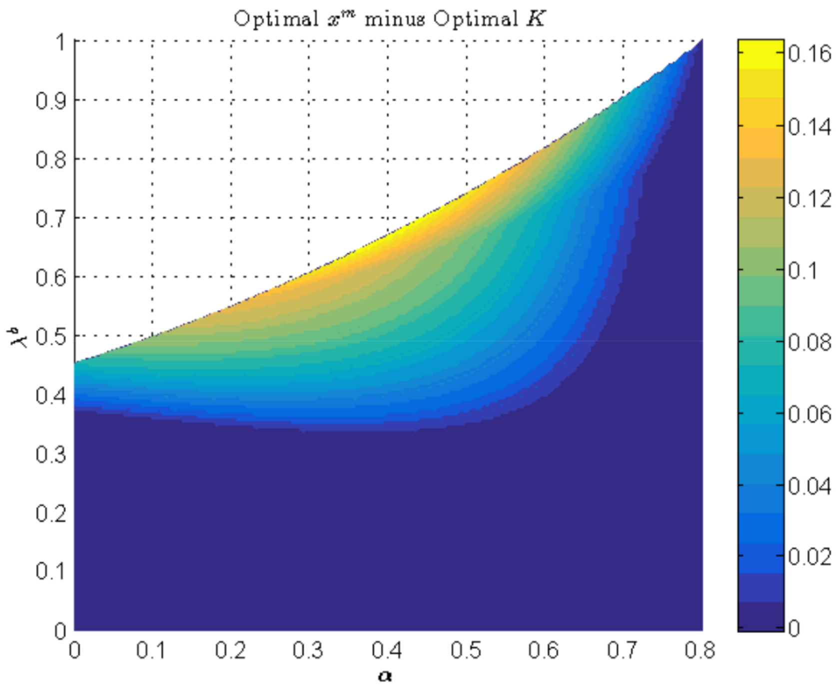

Figure 3: Values of $x^m - K$ with single-market search in endowment economy

that when $B > \alpha S$, it is possible that the intermediary attracts an excessive number of buyers to the middleman sector $x^m > K$, resulting in stockouts, in order to lower the wholesale price in the middleman sector.[16] When this occurs, the resource constraint is tight and the outside value of agents is high so that economizing on stocking costs is relatively important.

Multi-market search: With multi-market search technologies, the participation constraint of agents is not the issue but the intermediation fee and the middleman's price should be acceptable relative to the outside value. Hence, the intermediary faces the incentive constraints, (5) – (7) with an appropriate modification of the match probability in the D market (see the details in the proof of Proposition 5). As before, these conditions are reduced to $f \leq v(x^m, K)$.

To be consistent, we maintain the assumption of a zero trade share of the sellers, $\beta = 1$, in the D market. This assumption now implies that sellers are fully exploited in the C market, thus $p^w = W(x^s) = 0$ for any $x^s \geq 0$.

With the multi-market search setup, the buyers' outside option value depends positively on

[16] The figures in this subsection are drawn with $B = 0.8$ and $S = 1$. We cut out the region where negative profits result, for high values of λ^b.

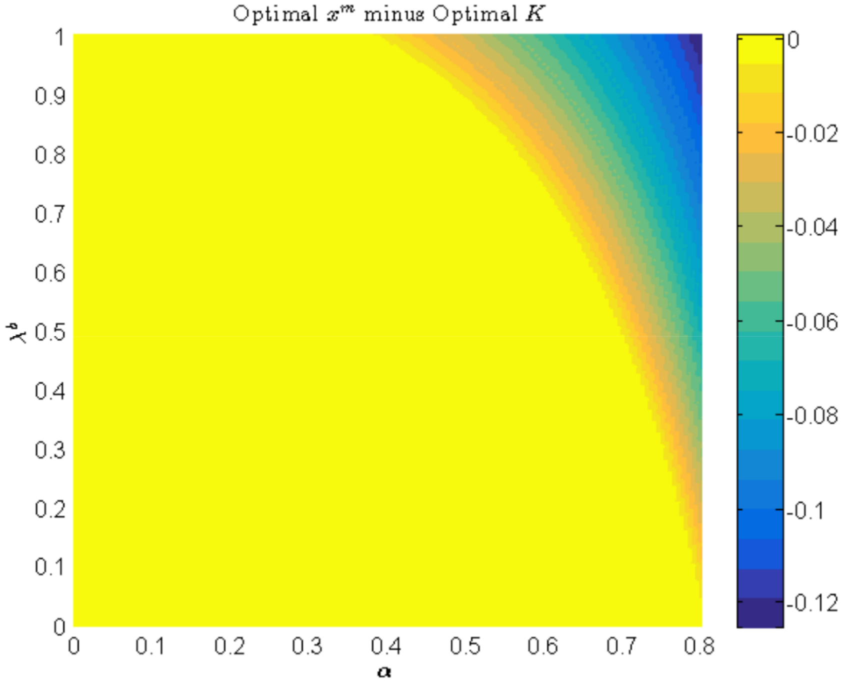

Figure 4: Values of $x^m - K$ with multi-market search in endowment economy

the number of sellers available in the D market. This has the following consequences. First, just as in the baseline setup, a pure middleman mode can never be profit maximizing. Second, in our endowment economy, the intermediary may wish to stock more inventories than the number of buyers visiting the middleman sector. This is because a larger K will crowd out the supply available in the D market, which will eventually lower the outside value of buyers and increase the profit. Therefore, unlike in all the previous setups, the solution here allows for an excess inventory in the middleman sector.

Proposition 5 *Consider the endowment economy outlined above with multi-market search technology, and the zero trade share of sellers in the D market. The intermediation chooses to be a market-making middleman or a pure market-maker with* $x^m \leq K = \alpha S$.

Proof. See the Appendix. ■

Figure 4 shows the occurrence of excess inventory holdings in the middleman sector with high values of λ^b and α. This confirms our intuition that the crowding-out effect of excess inventory is stronger when the agents outside value in the D market is higher.

Comparing Proposition 4 and 5, we can summarize the implication of search frictions in wholesale markets represented by α and the agents' search technologies in retail markets on the choice of intermediation mode in our endowment economy as follows.

- For $\alpha S \geq B$, the middleman can stock the full inventory that satisfies all the buyers' demand. As in the benchmark setup, the intermediary chooses to be a pure middleman with single-market search, but uses an active platform with multi-market search. Unlike in the previous setups, the middleman holds an excessive amount of inventory.

- For $\alpha S < B$, the full inventory is not possible due to the aggregate resource constraint. The intermediary uses a platform irrespective of whether agents search in a single or in multiple markets. Our main insight is still valid. Namely, *the intermediation mode is further away from the pure middleman mode when agents search in multiple markets, rather than in a single market.* The size of the middleman sector, measured by x^m, is smaller with multi-market search than with single-market search technologies.

4.3 Cost functions

Inventory Costs. In the baseline model, we assume zero inventory costs of the middleman. In this section, we consider the following convex inventory-cost function,

$$C'(K) \geq 0; \quad C''(K) \geq 0; \quad C(0) = 0; \quad C'(K) < 1 - \lambda^b.$$

The last condition guarantees that $C(B) < B(1 - \lambda^b)$. We assume $\beta = 1$ for simplicity. With positive inventory costs, it may be profitable to activate a platform even with single-market search. Still, we show that our main insight is valid.

As in the baseline model, profit maximization requires $K = x^m$. With single-market search, the problem of a market-making middleman can be described as

$$\max_{x^m, f} \Pi(x^m) = S(1 - e^{-x^s})f(x^s) + x^m p^m - C(x^m). \tag{20}$$

subject to $p^m = 1 - \lambda^b$ and $f(x^s) = 1 - \lambda^b e^{x^s}$. The first constraint is given by the buyers' participation constraint in the C market, i.e., $V^m = 1 - p^m \geq \lambda^b$, while the second constraint is given by the buyers' indifference condition, i.e., $V^m = V^s = e^{-x^s}(1 - f)$ (a formal derivation can be found in the Appendix): the platform fee $f = f(x^s)$ is strictly decreasing (increasing) in x^s (x^m). Intuitively, the tighter the platform, the lower the fee that the intermediary needs to offer, in order to make buyers indifferent between the platform and the middleman sector. The negative dependence of the platform fee on the platform size favors the middleman mode. The first order condition becomes

$$\frac{\partial\Pi(x^m)}{\partial x^m} = -e^{-x^s} + \lambda^b e^{x^s} + 1 - \lambda^b - C'(x^m) \equiv \Theta_{Sfoc}(x^m) = 0. \tag{21}$$

With multiple-market search, the objective function is the same as in (20), but the constraints are $p^m(x^s) = f(x^s) = 1 - \lambda^b e^{-x^s}$ (by (16) and $v(\cdot) = f$ as in Lemma 1). As before, the positive dependence of the middleman's price and the platform fee on the platform size favors the market-maker mode. Observe that

$$\begin{aligned} \frac{\partial\Pi(x^m)}{\partial x^m} &= (1 - e^{-x^s})(1 - 2\lambda^b e^{-x^s}) - \frac{\lambda^b x^m e^{-x^s}}{S} - C'(x^m) \quad (22) \\ &< (1 - e^{-x^s})(1 - 2\lambda^b e^{-x^s}) - C'(x^m) \\ &= \Theta_{Sfoc}(x^m) - \lambda^b \left[2e^{-x^s}(1 - e^{-x^s}) + e^{x^s} - 1\right] \\ &< \Theta_{Sfoc}(x^m), \end{aligned}$$

implying that the marginal profit of increasing the size of the middleman sector is smaller with multi-market search than with single-market search. The logic behind this is essentially the same as in the baseline model, and is generalized as follows.

Proposition 6 *Consider the convex inventory costs of a middleman defined above. Then, a platform is activated even under a single-market search technology. Still, the size of the platform with multi-market search is larger than or equal to that with single-market search.*

Proof. See the Appendix. ■

Prior production/purchase before joining the platform. In real-life markets, sellers sometimes need to prepare (produce or purchase) their product for sale prior to market entry. For example, online sellers find it important to display their product's image and keep it ready for delivery before actual transactions occur. A similar issue arises when asset holders are required to commit to their portfolio before trading with their brokers. In these situations, because sellers incur costs irrespective of their success in the platform, attracting sellers to the platform is costly and so the relative profitability of the market-maker mode is reduced. We show, however, that our insight remains valid in such a setting. Interestingly, we also find that a platform can be activated even when the net profit obtained from the platform business is negative.

The only modification that is required now is to introduce a participation constraint for sellers to operate on a platform. Under single-market search, this is irrelevant because the pure middleman mode remains profit maximizing. With multiple-market search, the participation constraint is given by

$$W(x^s) - f_p \geq c_E, \tag{23}$$

where $c_E \geq 0$ represents entry costs of sellers to the platform, $f_p \geq 0$ (or $f_p \leq 0$), a platform participation fee (or subsidy) to each individual seller, and $W(x^s) = \eta(x^s)(p^m - f)$ the equilibrium value of sellers who participate on the platform. With $\beta = 1$, i.e., zero payoff in the D market for sellers, the intermediary sets $f = p^m = 1 - \lambda^b e^{-x^s}$, satisfying the incentive constraint (6) (note that the participation in the D market does not require prior production/purchase as before), and $f_p = -c_E$. That is, the intermediary should subsidize the entry cost and fully extract the trade surplus in the platform. The profit of a market-making middleman is

$$\tilde{\Pi}(x^m) = S\left[(1 - e^{-x^s})f - c_E\right] + x^m p^m,$$

while the profit of a middleman is

$$\tilde{\Pi}(B) = B(1 - \lambda^b).$$

Comparing these profits, one can find a value of $x^m < B$ (e.g. imagine a neighbourhood of $x^m = B - Sx^s \approx B$) and $c_E > 0$ for which the platform profit is negative but $\tilde{\Pi}(x^m) > \tilde{\Pi}(B)$. This leads to the following result.

Proposition 7 *Suppose sellers incur production/purchasing costs prior to platform entry. Then, an active platform can be profit maximizing even when the platform entry cost is higher than the platform fee revenue.*

One benefit of having an active platform in the C market for the intermediary is to reduce competition so that it can set a higher price in the middleman sector. This benefit can be the major source of profits for market-making middlemen even when the platform-entry costs are so high that the net profit from the platform business is negative.

4.4 Competing intermediaries

Our framework can be extended to study competing intermediaries. We consider two intermediaries who make a simultaneous choice of platform fees and/or price of their good. In particular, we are interested in whether an active platform with positive fees of an incumbent intermediary, referred to as I, can be profitable when the other intermediary, referred to as E, enters with adopting a pure middleman mode or a pure market-maker mode. To simplify the analysis we abstract away from the decentralized market trade, and assume zero marginal costs and zero entry costs.

Single-market search: With single market search, irrespective of the intermediation mode of E (and beliefs of agents on which intermediary to be favorable), I has no strict incentive to activate a platform with positive fees. To see this, let V_E be the value of buyers to visit E. If I chooses to be a pure middleman then its profit is Bp^m with price $p^m = 1 - V_E$. If I activates a platform, then, just like in our benchmark setup, the fee should satisfy $f \leq 1 - V_E$ and so its maximum attainable profit with positive fees is strictly less than $B(1 - V_E)$. The intuition remains the same as before – with single market search, the middlemen mode is the way to achieve the highest trade surplus.

Multiple-market search with a pure middleman E To be consistent with the previous analysis, we assume that agents visit I prior to E by default, featuring that I is a well-established intermediary in the market, whereas E is a newcomer and has no regular customers in the beginning.

When E is a pure middleman with price $p_E \in [0, 1]$, I's price/fee (p^m, f) should satisfy the incentive constraints,

$$p^m \leq p_E \quad \text{and} \quad f \leq p_E,$$

respectively. The major difference from the benchmark is that, as a pure middleman, E would undercut any positive price/fee of I and so an active platform with positive fees can never survive.

Multiple-market search with a pure market-maker E When E is a pure market-maker with a fee f_E, the incentive constraints become

$$p^m \leq 1 - V_E(f_E) \quad \text{and} \quad f \leq 1 - V_E(f_E).$$

E could either act as a "second source" for intermediation service, or undercut I and be the "sole source".

When E operates as the second source, he adjusts f^E considering how that affects the participating buyers and sellers. Surprisingly, the transaction fee of E affects the intermediation mode of I. To see this, notice that a lower f^E means a better outside option for a buyer. The buyer now finds it more attractive to visit an individual seller in I rather than the middleman, since even if the buyer in question remains unmatched at I, the outside option to trade at E (with a lower f^E) is a favorable prospect. As such, within I more buyers switch from the frictionless middleman to the frictional platform, leaving more unmatched buyers and less unmatched sellers joining E. Ultimately, the trade-off between more participating buyers (by decreasing f^E) and more participating sellers (by increasing f^E) leads to an $f^E < 1$ in equilibrium.

When E operates as the sole source, the undercutting would not be complete — since E will not be able to accommodate all buyers, there will exist residual demands left to I.

In either case, buyers have a positive outside option, an active platform can better exploit the intermediated surplus as we presented in the benchmark model. Hence, using an active platform with positive fees can be a profitable business mode for an intermediary when the other intermediary also activates a platform.

Proposition 8 *Consider two competing intermediaries, one is an incumbent (I), just like our original intermediary, and the other is an entrant (E) that replaces the D market. Then, I activates a platform with positive fees only when agents search in multiple markets and E also adopts an active platform.*

Proof. See the Appendix. ■

5 Examples

Our analysis shows that a marketmaking middleman is more likely to emerge with multi-market search technologies than with single-market technologies. In this section, we offer some real market examples.

Online retailers. The electronic commerce company Amazon.com is traditionally an online retailer, who mainly aims at selling its inventories to customers. In the late 1990s, Amazon was facing fierce competition from local brick and mortar rivals, as well as chain stores such as Walmart, Sears, etc., and especially from eBay. According to the book, *The Everything Store: Jeff Bezos and the Age of Amazon*, Jeff Bezos worried that eBay may become the leading online retailer who attracts the majority of customers. In the summer of 1998, he invited eBay's management team and suggested the possibility of a joint venture or even of buying out their business. This is perhaps Amazon's first attempt to set up an online marketplace. In the end, this trial failed. After several more trials and errors, however, Amazon finally launched their own marketplace in the early 2000s. The entry version of our model in Section 5.3 captures well Amazon's reaction to the entry by eBay.

Amazon's launch of the platform business influenced significantly the book industry. On the one hand, Amazon attracts many of its competitors to join their platform. Indeed, Amazon drove physical book and record stores out of business, and many bookstore owners re-launched their business on the Amazon-website platform. On the other hand, Amazon lowers the chance of buyers to trade outside. As local bookstores disappeared, it became the habit for most book buyers to start their everyday online-shopping using Amazon as the prime site De los Santos et al. (2012). Overall, these observed phenomena are in line with our theory.[17][18] Not surprisingly, Amazon promoted this shopping pattern to customers in other product categories.

The general picture of the online travel agency industry is similar. Before the rise of Internet, most intermediaries in this industry acted as a pure middleman. In the middleman mode, hotels sell rooms to a middleman in bulk at discounted prices. The middleman then sells them to customers at a markup price. With the online reservation system, a market-making mode became popular, wherein hotels pay a market maker (e.g. Booking.com) commission fees upon successful reservations. The hotels post their services and prices on the platform. Expedia used to be a pure middleman but is nowadays a representative market-making middleman who employs both of these intermediation modes.

Specialist markets. The New York Stock Exchange (NYSE) is a specialist market, which is defined as a hybrid market that includes an auction component (e.g., a floor auction or a limit

[17] Nowadays, most buyers and sellers use Amazon as the main website (the first one to visit). On the seller side, according to a survey on Amazon sellers conducted in 2016, more than three-quarters of participants sell through multiple channels, online marketplaces, webstores and bricks-and-mortar stores. The second most popular channel, after Amazon, is eBay, with 73% selling through this marketplace. On the buyer side, according to a recent Reuters/Ipsos poll, 51 percent of consumers plan to do most of their shopping on the Amazon.com.

[18] An alternative (or complementary) to our theory would be a product selection story where Amazon uses the platform for third-party sellers to add new products with the demands too small for Amazon to offer. Once a product is "tested" to be popular enough, Amazon starts to also offer it through the middleman sector. This would be certainly a valid explanation but by far not the exclusive one. First, if this explanation were correct, we should eventually observe that most popular products are listed by Amazon, and most not-so-popular products are listed by independent sellers. In reality, however, many high-demand products are listed by both Amazon and third-party sellers at the same time, and importantly, they are competing with each other. This competition goes against the proposed explanation, but is more in line with our theory. In fact, Amazon could avoid fierce competition with strong competitors operating in the Amazon marketplace, such as GreenCupboards or independent sellers who own 'Buy Boxes', by giving up dealing with such a product in the middleman sector, which should in turn increase their fee revenue.

order book) together with one or more specialists (also called designated market makers). The specialists have some responsibility for the market: as brokers, they pair executable customer orders; and as dealers, they post quotes with reasonable depth Conroy and Winkler (1986).

As for their role as dealers in the exchanges, our model suggests that, at least for less active securities (represented by smaller outside option values in the model), the specialists' market can provide predictable immediacy and increase the trading volume and liquidity. This is consistent with the trend to adopt hybrid markets in derivative exchanges and stock markets around the world, especially for thinly-traded securities. For example, several European stock exchanges implemented a program which gives less active stocks an option of accompanying a designated dealer in the auction market. These initiatives were effective not only in enhancing the creation of hybrid specialist markets, but also in increasing trade volumes and reducing liquidity risks (Nimalendran and Petrella 2003, Anand et al. 2009, Menkveld and Wang 2013, and Venkataraman and Waisburd 2007.)

Another prediction from our analysis is related to the changing competitive environment faced by securities exchanges. As a broader implication, our result that the increased outside pressure goes hand in hand with more decentralized trades, captures the background trend in general: the market for NYSE-listed stocks was highly centralized in the year of 2007 with the NYSE executing 79% of volume in its listings; in 2009, this share dropped to 25% (Securities and Exchange Commission 2010); today, the order-flow in NYSE-listed stocks is divided among many trading venues – 11 exchanges, more than 40 alternative trading systems, and more than 250 broker-dealers in the U.S. (Tuttle 2014). As a more specific implication, we show that the increased pressure from outside markets will scale up the platform component. This is indeed the case. Starting from 2006, the NYSE adopted the new hybrid trading system featuring an expanded platform sector "NYSE Arca", which allows investors to choose whether to trade electronically or by using traditional floor brokers and specialists. The new system is further supplemented by several dark pools, akin to platforms, owned by the NYSE. These strategies are also adopted by NASDAQ which has been thought of as a typical dealers' market. In addition, the use of fees is widely adopted, as is consistent with our theory. For instance, in 2014, the NYSE offered banks a discount of trading costs by more than 80% conditional on their agreement to stay away from the outside dark pools and other off-exchange venues.[19]

[19] See a report "NYSE Plan Would Revamp Trading" in the Wall Street Journal, 2014. http://www.wsj.com/articles/intercontinental-exchange-proposing-major-stock-market-overhaul-1418844900.

Real estate agencies. While intermediaries in housing markets are mostly thought of as brokers, i.e., platforms, the business mode employed by the Trump family is a marketmaking middleman. The Trump Organization holds several hundred thousand square feet of prime Manhattan real estate in New York City (NYC) and some more in other big cities. Besides developing and owning residential real estate, the Trump family operates a brokerage company that deals with luxury apartments, the Trump International Realty. Both of these companies target the same market in NYC. Indeed, the Trump's business mode is a marketmaking middleman – both owning his own residential towers, and offering broker services. According to Forbes, the latter portion of Trump's empire becomes by far his largest business with a valuation of 562 million in 2006. Another example is Thor Equities, a large-scale real estate company, which owns and redevelops retail properties in Soho, Madison Avenue, and Fifth Avenue, and also runs brokerage agencies, Thor Retail Advisors and Town Residential.

In the endowment economy version of our model, we show that the marketmaking middleman over-invests in inventory with multi-market search, up to the point where the resource constraint is binding. Perhaps, the real estate market in NYC is an appropriate example of this since it is well known to be competitive and tight for house/apartment hunters. In addition, most new developments in big cities are renovations of old houses, and so we can roughly regard the total supply as fixed. Notably, top real estate firms in NYC attempt to expand their business by being engaged in many new joint projects with developers. Mapped into our model, these efforts are aimed at relaxing their resource constraint and increasing their inventory. For example, Nest Seekers, a real estate brokerage and marketing firm in NYC, works tightly with constructors on new developments. They work together from the very early stage of layout design and fund raising (in some cases Nest Seekers offers their own capital) to the later marketing stage. Nest Seekers provides qualified sales and administrative staff to the sales office, prepares pricing schedules, manages all contracts with the brokerage community, and is eventually in charge of the entire marketing process. This co-development business is one step beyond the middleman mode formulated in our theory, but is considered as an alternative way to secure their inventory.[20] This business mode is adopted in many other big real-estate companies in NYC, such as Douglas Elliman, Stribling,

[20] Strictly speaking, Nest Seekers does not own properties, but becomes the exclusive agent of projects. So far, they have co-developed/marketed more than 30 projects. See https://www.nestseekers.com/NewDevelopments. A report titled "Inside the fight for Manhattans most valuable new development exclusives" by The Real Deal introduces more detailed information on how brokers cooperate with developers, which is available in http://therealdeal.com/2016/03/15/inside-the-fight-for-manhattans-most-valuable-new-development-exclusives/ (visited on July 15, 2016).

and Corcoran.

Finally note that some intermediaries do not help to promote market new developments, but also manage apartment complexes, which constitutes another source of "inventory". For example, Brown Harris Stevens provides the residential management service for its customers since cooperative apartments were first introduced to NYC. These cooperative apartments usually contain hundreds of units in one building, and Brown Harris Stevens is then in charge of listing these properties when they are for rent or on sale.

6 Empirical evidence

The model's predictions on the choice of intermediation mode can be empirically tested. As in the last extension of competing intermediaries, we take Amazon as the centralized market and eBay as the decentralized market. Our model predicts that Amazon is more likely to sell the product as a middleman when the chance of buyers to meet a seller in the decentralized market, λ^b, is low, the buyers' bargaining power β is low, and the total demand B is high. That is,

$$\Pr(\text{Amazon's middlemen mode is active}) = f(\underset{-}{\lambda^b}, \underset{-}{\beta}, \underset{+}{B}),$$

where $-$ $(+)$ indicates a negative (positive) correlation.

We collected data from *www.amazon.com* and *www.ebay.com*, focussing on one product category, namely pans. We think this product choice is appropriate not only because there are many observations for both eBay and Amazon but also because pans require some minimum consideration and search before a purchase decision is made. In addition, we analyze the theoretically most relevant case where Amazon acts as a marketmaking middleman for this product category: For 32% of the sample, Amazon acts as a middleman; for the other 68%, Amazon acts as a market maker. Our data matches each product on sale at Amazon to a list of the offers at eBay. Using the information on individual prices and sellers, we construct proxies for key parameters in the model: λ^b, β and B as explained below (see the Web Appendix for more details). Since our data are not experimental, our evidence should be interpreted as suggestive rather than causal.

Table 1 summarizes various cross-sectional regressions of Amazon's intermediation mode, represented by *sellByAmazon*, which is a dummy variable that takes a value of 1 if the product is sold by Amazon. It is an indicator that Amazon is an active middleman for that product. Other

Table 1: Regressions for Amazon's intermediation mode

	(1) Linear	(2) Linear	(3) Probit	(4) Probit
sellersEbayRelative	-0.00630***		-0.00778***	
	(0.000765)		(0.00119)	
sellersEbayRefined		-0.00159**		-0.00156*
		(0.000595)		(0.000698)
sellersAmazon		-0.000552		-0.000669
		(0.000917)		(0.000996)
log(rank)	-0.103***	-0.107***	-0.105***	-0.110***
	(0.00442)	(0.00463)	(0.00500)	(0.00517)
priceDiff	0.105***	0.111***	0.124***	0.133***
	(0.00820)	(0.00830)	(0.0107)	(0.0111)
log(price)	0.0390***	0.0406***	0.0484***	0.0510***
	(0.00608)	(0.00613)	(0.00680)	(0.00688)
listedDays	0.0602***	0.0660***	0.0717***	0.0784***
	(0.00480)	(0.00486)	(0.00599)	(0.00604)
Observations	6457	6457	6457	6457
Adjusted R^2	0.136	0.130		

Note: Columns (1) and (2) use linear probability model, and columns (3) and (4) use probit model. For probit models, the marginal effects evaluated at the sample mean are reported. *sellersEbayRefined* is the number of sellers on a refined list of sellers on eBay by matching the title of offers with the Amazon product title and restricting the price of offers between 0.5 and 1.5 times the Amazon price. *sellersAmazon* is the number of third-party sellers on Amazon. Robust standard errors are reported in parentheses. Other variables are explained in the main text. * denotes $p < 0.05$, ** denotes $p < 0.01$, *** denotes $p < 0.001$.

variables we use for the linear regression in Column 1 are discussed below. *sellersEbayRelative* is a proxy for λ^b and is defined as the number of sellers on eBay divided by the number of third-party sellers on Amazon. *rank* is a proxy for total demand B and is defined a the sales rank within the product category. Rankings are negatively correlated with sales (e.g., a product with a rank value of 100 is associated with more sales than a product with a rank value 200). *priceDiff* is a proxy for β and is defined as the log of the median price of eBay offers minus the log of the Amazon price. *listedDays* controls for the number of days since the product was first listed on Amazon. Our theoretical model predicts that *sellByAmazon* is negatively correlated to *sellersEbayRelative*, negatively correlated to *log(rank)* and positively correlated to *priceDiff*. As shown in Table 1, all explanatory variables have the expected signs and are statistically significant in all the specifications (including those where we use alternative proxies and probit regressions).[21] To quantify the effect of available options in eBay, we find that the chance that Amazon acts as a middleman decreases by 3.7 percent for a one-standard deviation increase in *sellersEbayRelative* (λ^b), and increases by 0.1 percent for a one percent increase in the median eBay price relative to the Amazon price (proxied by *priceDiff*, β). In the Web Appendix, we give more detailed information on the data and we experiment with a number of different specifications, but none of them alters our main results.

7 Conclusion

This paper developed a model in which market structure is determined endogenously by the choice of intermediation mode. We considered two representative business modes of intermediation that are widely used in real-life markets: a market-making mode and a middleman mode. We derived conditions for a mixture of the two modes, a *marketmaking middleman* to emerge.

One implication of our theory is that intermediaries can use a platform to reduce competition with sellers in the decentralized market. However, this is done by inducing consumers to search excessively and so generates inefficiencies. For future research, it would be interesting to examine this issue from the viewpoint of a regulator.

[21] Recent work by Zhu and Liu (2018) also examines empirically the product choice by Amazon. While their approach is very different from ours, it is interesting to note that both share some common evidence. For instance, we find that the number of sellers on Amazon is negatively associated with the likelihood of Amazon to act as a middleman. This may reflect a crowding out effect of Amazon on third-party sellers. Similarly, Zhu and Liu (2016) find that Amazon's entry could discourage third-party sellers and eventually force them to leave the platform. Also, our evidence suggests that Amazon is more likely to sell more established products with higher prices. This is also consistent with Zhu and Liu (2018)'s findings that Amazon may be targeting on successful products to exploit the surplus from third-party sellers.

Appendix

Proof of Lemma 1

Using $K \leq x^m$ and (16), the intermediary's problem can be written as

$$\begin{aligned}\max_{x^m,f,K} \Pi(x^m,f,K) &= S(1-e^{-x^s})f + \min\{K,x^m\}p^m - Kc \\ &= S(1-e^{-\frac{B-x^m}{S}})f + K(1-\lambda^b e^{-\frac{B-x^m}{S}}\beta)(1-c) - x^m e^{-\frac{B-x^m}{S}}(v(x^m,K)-f)\end{aligned}$$

subject to (13) and

$$0 < K \leq x^m < B.$$

Observe that: $\lim_{x^m \to B} \Pi(x^m,f,K) = \tilde{\Pi}(B)$ and $\lim_{x^m \to 0} \Pi(x^m,f,K) = \tilde{\Pi}(0)$, where $\tilde{\Pi}(B) = B(1-\lambda^b\beta)(1-c)$ is the profit for the pure middleman mode (14) and $\tilde{\Pi}(0) = S(1-e^{-\frac{B}{S}})f$ is the profit for the pure market-maker mode (15). Hence, we can compactify the constraint set and set up a general problem to pin down a profit-maximizing intermediation mode using the following Lagrangian:

$$\mathcal{L} = \Pi(x^m,f,K) + \mu_k(x^m-K) + \mu_b(B-x^m) + \mu_v(v(x^m,K)-f) + \mu_0 K,$$

where the μ's ≥ 0 are the lagrange multiplier of each constraint. In the proof of Proposition 2, we show that the following first order conditions are necessary and sufficient:

$$\frac{\partial \mathcal{L}}{\partial x^m} = \frac{\partial \Pi(x^m,f,K)}{\partial x^m} + \mu_k - \mu_b + \mu_v \frac{\partial v(x^m,K)}{\partial x^m} = 0, \quad (24)$$

$$\frac{\partial \mathcal{L}}{\partial f} = \frac{\partial \Pi(x^m,f,K)}{\partial f} - \mu_v = 0, \quad (25)$$

$$\frac{\partial \mathcal{L}}{\partial K} = \frac{\partial \Pi(x^m,f,K)}{\partial K} - \mu_k + \mu_0 + \mu_v \frac{\partial v(x^m,K)}{\partial K} = 0. \quad (26)$$

The solution is characterized by these and the complementary slackness conditions of the four constraints.

We now prove the claims in the lemma. First, (25) implies that we must have

$$\mu_v = S(1-e^{-x^s}) + x^m e^{-x^s} > 0,$$

which implies the binding constraint (13),

$$f = v(x^m,K) = \left[1-\lambda^b e^{-\frac{B-x^m}{S}}\beta - \lambda^s\left\{1-\frac{K}{B}-\frac{S}{B}(1-e^{-\frac{B-x^m}{S}})\right\}(1-\beta)\right](1-c).$$

Second, applying μ_v from (25) into (26) gives

$$\mu_k = \left[1-\lambda^b e^{-\frac{B-x^m}{S}}\beta + \lambda^b\left(1-e^{-\frac{B-x^m}{S}}\right)(1-\beta)\right](1-c) + \mu_0 > 0,$$

which implies that $K = x^m$. This completes the proof of Lemma 1. ■

Proof of Proposition 2

⊙ Active platform. First of all, we show that the platform will always be active (i.e., $x^m < B$) in equilibrium. Substituting μ_k, μ_v into (24),

$$\begin{aligned}(1-c)^{-1}(\mu_b-\mu_0) &= -e^{-\frac{B-x^m}{S}}\left[1-\lambda^b e^{-\frac{B-x^m}{S}}\beta - \lambda^b\left\{\frac{B}{S}-(1-e^{-\frac{B-x^m}{S}})\right\}(1-\beta)\right] \quad (27) \\ &\quad -\lambda^b\frac{x^m}{S}e^{-\frac{B-x^m}{S}} + 1 - \lambda^b\beta + \lambda^b(1-e^{-\frac{B-x^m}{S}})^2 \\ &\equiv \phi(x^m \mid B,S,\beta,\lambda^b).\end{aligned}$$

Suppose that the solution is $x^m = B$. Then, (27) yields $\phi(B \mid \cdot) = (1-c)^{-1}\mu_b = -\frac{B}{S}\lambda^b\beta < 0$, which contradicts $\mu_b \geq 0$. Hence, the solution must satisfy $x^m < B$ (which implies $\mu_b = 0$).

⊙ Market-making middleman or pure market-maker. Second, we derive the condition for a pure market-maker $x^m = 0$ or a market-making middleman $x^m > 0$. Since $\phi(B \mid \cdot) < 0$, if $\phi(0 \mid \cdot) > 0$, there exists $x^m \in (0, B)$ that satisfies $\phi(x^m \mid \cdot) = 0$, i.e., a market-making middleman. Further,

$$\frac{\partial \phi(x^m \mid \cdot)}{\partial x^m} \mid_{\phi=0} = -\frac{1}{S}\left[1-\lambda^b\beta+\lambda^b(1-e^{-\frac{B-x^m}{S}})2\lambda^b(1-e^{-\frac{B-x^m}{S}})e^{-\frac{B-x^m}{S}}\right]-\frac{\lambda^b}{S}e^{-\frac{B-x^m}{S}}(1-e^{-\frac{B-x^m}{S}})<0.$$

This implies that the allocation of the middleman sector $x^m \in (0, B)$ is unique (if it exists), and that if $\phi(0 \mid \cdot) < 0$ then $\phi(x^m \mid \cdot) < 0$ for all $x^m \in [0, B]$ and the solution must be a pure market maker, $x^m = 0$.

Now, we need to investigate the sign of it:

$$\begin{aligned}\phi(0 \mid B, S, \beta, \lambda^b) &= -e^{-x}\left[1-\lambda^b e^{-x}\beta-\lambda^b\left(x-1+e^{-x}\right)(1-\beta)\right]+1-\lambda^b\beta+\lambda^b(1-e^{-x})^2 \\ &\equiv \Theta(x),\end{aligned}$$

where $x \equiv \frac{B}{S}$. Observe that:

$$\Theta(0) = 0 < 1-\lambda^b\beta+\lambda^b = \Theta(\infty),$$

and

$$\frac{\partial\Theta(x)}{\partial x} = e^{-x}\left[1-\lambda^b x+\lambda^b\beta(x-2)+4\lambda^b(1-e^{-x})\right].$$

This derivative has the following properties: $\frac{\partial\Theta(x)}{\partial x} \mid_{x=0} = 1-2\lambda^b\beta$;

$$\frac{\partial\Theta(x)}{\partial x} \mid_{\Theta(x)=0} = 1-\lambda^b\beta(1+e^{-x})+\lambda^b(1-e^{-x})(1+2e^{-x}) \equiv \Upsilon(x).$$

There are two cases.

- When $\lambda^b\beta \leq \frac{1}{2}$, we have $\frac{\partial\Theta(x)}{\partial x} \mid_{x=0} \geq 0$ and $\frac{\partial\Theta(x)}{\partial x} \mid_{\Theta(x)=0} > 0$, implying that no $x \in (0, \infty)$ exists such that $\Theta(x) = 0$. Hence, $\Theta(x) = \phi(0 \mid \cdot) > 0$ for all $x \in (0, \infty)$.
- When $\lambda^b\beta > \frac{1}{2}$, we have $\frac{\partial\Theta(x)}{\partial x} \mid_{x=0} < 0$. Hence, there exists at least one $\bar{x} \in (0, \infty)$ such that $\Theta(x) < 0$ for $x < \bar{x}$ and $\Theta(x) \geq 0$ for $x \geq \bar{x}$. Below we show that such a value has to be unique. For this purpose, observe that:

  $$\Upsilon(0) = 1-2\lambda^b\beta < 0 < 1+\lambda^b(1-\beta) = \Upsilon(\infty), \frac{\partial\Upsilon(x)}{\partial x} = \lambda^b e^{-x}(4e^{-x}-1+\beta),$$
  $$\frac{\partial\Upsilon(x)}{\partial x} \mid_{x=0} = \lambda^b(3+\beta) > 0, \frac{\partial^2\Upsilon(x)}{\partial x^2} \mid_{\frac{\partial\Upsilon(x)}{\partial x}=0} = -4e^{-x}\lambda^b e^{-x} < 0.$$

  These properties imply that there exists an $x' \in (0, \infty)$ such that $\Upsilon(x) <$ for all $x < x'$ and $\Upsilon(x) \geq 0$ for all $x \geq x'$. This implies that $\bar{x}$ is unique.

To summarize, we have shown that if $\lambda^b\beta \leq \frac{1}{2}$ then the solution is a market-making middleman $x^m \in (0, B)$ for all $x = \frac{B}{S} \in (0, \infty)$. If $\lambda^b\beta > \frac{1}{2}$ then there exists a unique critical value $\bar{x} \in (0, \infty)$ such that the solution is a market-making middleman for $x \geq \bar{x}$ and is a pure market-maker $x^m = 0$ for $x < \bar{x}$.

⊙ Second order condition. Finally, we verify the second order condition. Define $\mathbf{X} \equiv [x^m, f, K]$ and write the binding constraints as

$$h_1(\mathbf{X}) = v(x^m, K) - f,\ h_2(\mathbf{X}) = x^m - K.$$

The solution characterized above is a maximum if the Hessian of $\mathcal{L}$ with respect to $\mathbf{X}$ at the solution denoted by $(\mathbf{X}^*, \mu^*)$ is negative definite on the constraint set $\{\mathbf{w} : \mathbf{Dh}(\mathbf{X}^*)\mathbf{w} = 0\}$ with $\mathbf{h} \equiv [h_1(\mathbf{X}), h_2(\mathbf{X})]$. This can be verified by using the bordered Hessian matrix, denoted by H.

$$H \equiv \begin{bmatrix} 0 & D\mathbf{h}(\mathbf{X}^*) \\ D\mathbf{h}(\mathbf{X}^*)^T & D^2_{\mathbf{X}}\mathcal{L}(\mathbf{X}^*,\mu^*) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \frac{\partial h_1}{\partial x^m} & \frac{\partial h_1}{\partial f} & \frac{\partial h_1}{\partial K} \\ 0 & 0 & \frac{\partial h_2}{\partial x^m} & \frac{\partial h_2}{\partial f} & \frac{\partial h_2}{\partial K} \\ \frac{\partial h_1}{\partial x^m} & \frac{\partial h_2}{\partial x^m} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial f \partial x^m} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial K \partial x^m} \\ \frac{\partial h_1}{\partial f} & \frac{\partial h_2}{\partial f} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m \partial f} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial f^2} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial K \partial f} \\ \frac{\partial h_1}{\partial K} & \frac{\partial h_2}{\partial K} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial f \partial K} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial K^2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -\frac{\lambda^b}{S} e^{-x^s}(1-c) & -1 & \frac{\lambda^s}{B}(1-\beta)(1-c) \\ 0 & 0 & 1 & 0 & -1 \\ -\frac{\lambda^b}{S} e^{-x^s}(1-c) & 1 & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{x^m}{S} e^{-x^s} & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} \\ -1 & 0 & \frac{x^m}{S} e^{-x^s} & 0 & 0 \\ \frac{\lambda^s}{B}(1-\beta)(1-c) & -1 & \frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} & 0 & 0 \end{bmatrix}$$

with

$$\frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} = -\frac{1}{S} e^{-x^s} v + \left(-\frac{1}{S}\frac{x^m}{S}\lambda^b e^{-x^s}\beta + 2\left(1+\frac{x^m}{S}\right) e^{-x^s}\frac{\lambda^b}{S} e^{-x^s} - \frac{\lambda^b}{S}\left(1-e^{-x^s}\right) e^{-x^s} \right)(1-c),$$

$$\frac{\partial^2 \mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} = -\left(\frac{\lambda^b}{S} e^{-x^s}\beta + \left(1+\frac{x^m}{S}\right) e^{-x^s}\frac{\lambda^s}{B}(1-\beta) \right)(1-c).$$

The determinant is given by $|H| = -\frac{1}{S}\left[e^{-x^s} v(x^m, K^*) + \frac{x^m}{S}\lambda^b e^{-x^s}\beta(1-c) + 3\lambda^b e^{-x^s}\left(1-e^{-x^s}\right)(1-c)\right] < 0$. Thus, the sufficient condition is satisfied. This completes the proof of Proposition 2. ■

Proof of Corollary 1

In (27), we have:

$$\frac{\partial \phi(x^m \mid .,.,\beta,.)}{\partial \beta}\Big|_{(\phi(x^m|\cdot)=0)} = -\lambda^b(1-e^{-2x^s}) - \lambda^b e^{-x^s}\left(\frac{B}{S} - 1 + e^{-x^s}\right) < 0,$$

$$\frac{\partial \phi(x^m \mid B,.,.,.)}{\partial B}\Big|_{(\phi(x^m|\cdot)=0)} = \frac{1}{S}\left[1 + \lambda^b(1-\beta) - \lambda^b e^{-2x^s} + \lambda^b e^{-x^s}(1-e^{-x^s})\right] > 0$$

$$\frac{\partial \phi(x^m \mid .,S,.,.)}{\partial S}\Big|_{(\phi(x^m|\cdot)=0)} = -\frac{x^s}{S}\left[1 + \lambda^b(1-\beta) - \lambda^b e^{-2x^s} + \lambda^b e^{-x^s}\left(\frac{B}{x^s} - e^{-x^s}\right)\right] < 0$$

$$\frac{\partial \phi(x^m \mid .,.,.,\lambda^b)}{\partial \lambda^b}\Big|_{(\phi(x^m|\cdot)=0)} = -\frac{1-e^{-x^s}}{\lambda^b} < 0.$$

Hence, since $\frac{\partial \phi(x^m|\cdot)}{\partial x^m}\big|_{(\phi(x^m|\cdot)=0)} < 0$ (see the proof of Proposition 2), it follows that: $\frac{\partial x^m}{\partial \beta} < 0$; $\frac{\partial x^m}{\partial B} < 0$; $\frac{\partial x^m}{\partial S} > 0$; $\frac{\partial x^m}{\partial \lambda^b} < 0$. This completes the proof of Corollary 1. ■

Proof of Proposition 3

The proof takes steps that are very similar to the ones we made in the proof of Proposition 1. With the non-linear matching function, the intermediary's profit function is modified to

$$\begin{aligned} \Pi(x^m, f, K) &= S(1-e^{-x^s})f + \min\{K, x^m\}p^m \\ &= S(1-e^{-\frac{B-x^m}{S}})f + K(1-\lambda^b(x^D)\beta) - x^m e^{-\frac{B-x^m}{S}}(v(x^m,K) - f), \end{aligned}$$

where $x^D = \frac{\max\{B - \min\{x^m, K\} - S(1-e^{-x^s}), 0\}}{Se^{-x^s}}$, and the surplus function to

$$v(x^m, K) = 1 - \lambda^b(x^D)\beta - \lambda^s(x^D)(1-\beta).$$

With these profit and surplus functions, the constraints and the Lagrangian remain unchanged, and the first orders are given by (24) – (26) (the second order conditions are presented below). As before, (25) implies that we must have

$$\mu_v = S(1 - e^{-x^s}) + x^m e^{-x^s} > 0,$$

and the binding constraint (13). Further, substituting μ_v from (25) into (26) gives

$$\mu_k = \mu_0 + 1 - \lambda^b(x^D)\beta + \frac{K}{Se^{-x^s}}\lambda^{b\prime}(x^D)\beta + \frac{1 - e^{-x^S}}{e^{-x^s}}\left(\lambda^{b\prime}(x^D)\beta + (\lambda^b(x^D) + x^D\lambda^{b\prime}(x^D))(1 - \beta)\right). \quad (28)$$

Substituting μ_k, μ_v into (24) gives,

$$\begin{aligned}\mu_b &= \mu_0 - e^{-x^s}\left(1 - \lambda^b(x^D)\beta - \lambda^b(x^D)x^D(1 - \beta)\right) + 1 - \lambda^b(x^D)\beta + \frac{B - K}{S}\frac{K}{Se^{-x^s}}\lambda^{b\prime}(x^D)\beta \\ &\quad + \frac{B - K}{S}\frac{1 - e^{-x^S}}{e^{-x^s}}\left(\lambda^{b\prime}(x^D)\beta + (\lambda^b(x^D) + x^D\lambda^{b\prime}(x^D))(1 - \beta)\right). \quad (29)\end{aligned}$$

Suppose now that $x^m = B$ and $K > 0$. Then, $\mu_k > 0$ in (28) if and only if

$$1 - \lambda^b(x^D)\beta + \frac{K}{S}\lambda^{b\prime}(x^D)\beta > 0,$$

and $\mu_b \geq 0$ in (29) if and only if

$$\frac{B - K}{S}\left[(1 - \beta)\lambda^b(x^D) + \frac{K}{S}\lambda^{b\prime}(x^D)\beta\right] \geq 0,$$

with $x^D = \frac{B-K}{S}$. Both of these conditions are satisfied only when $K = B$ (which implies $x^D = 0$, satisfying the latter condition) and

$$1 - \lambda^b(0)\beta + \frac{B}{S}\lambda^{b\prime}(0)\beta > 0 \quad (30)$$

(satisfying the former condition with $x^D = 0$). Under this condition, the solution is unique, $K = B = x^m$, $x^s = 0$ and $f = v(B, B)$. Hence, we have shown that the solution can be a pure middleman $x^s = 0$ only if (30) holds and otherwise the solution must be $x^s > 0$ (either a marketmaking middleman or a pure marketmaker).

Finally, we verify the second order condition. With the modified profit and surplus functions, as before, the bordered Hessian matrix is given by

$$\begin{aligned} H &\equiv \begin{bmatrix} 0 & D\mathbf{h}(\mathbf{X}^*) \\ D\mathbf{h}(\mathbf{X}^*)^T & D^2_{\mathbf{X}}L(\mathbf{X}^*, \mu^*) \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & \frac{\partial h_1}{\partial x^m} & \frac{\partial h_1}{\partial f} & \frac{\partial h_1}{\partial K} \\ 0 & 0 & \frac{\partial h_2}{\partial x^m} & \frac{\partial h_2}{\partial f} & \frac{\partial h_2}{\partial K} \\ \frac{\partial h_1}{\partial x^m} & \frac{\partial h_2}{\partial x^m} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial f \partial x^m} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial K \partial x^m} \\ \frac{\partial h_1}{\partial f} & \frac{\partial h_2}{\partial f} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial f} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial f^2} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial K \partial f} \\ \frac{\partial h_1}{\partial K} & \frac{\partial h_2}{\partial K} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial f \partial K} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial K^2} \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & \frac{\partial v(\mathbf{X}^*)}{\partial x^m} & -1 & \frac{\partial v(\mathbf{X}^*)}{\partial K} \\ 0 & 0 & 1 & 0 & -1 \\ \frac{\partial v(\mathbf{X}^*)}{\partial x^m} & 1 & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{B}{S} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} \\ -1 & 0 & \frac{B}{S} & 0 & 0 \\ \frac{\partial v(\mathbf{X}^*)}{\partial K} & -1 & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} & 0 & 0 \end{bmatrix} \end{aligned}$$

with

$$\begin{aligned} \frac{\partial^2 L(\mathbf{X}^*, \mu^*)}{\partial x^{m2}} &= -\frac{1}{S}f - B\frac{\partial^2 \lambda^b}{\partial x^{m2}}(\mathbf{X}^*)\beta - (2 + \frac{B}{S})\frac{\partial v(\mathbf{X}^*)}{\partial x^m}, \\ \frac{\partial^2 L(\mathbf{X}^*, \mu^*)}{\partial x^m \partial K} &= -\frac{\partial \lambda^b}{\partial x^m}\beta - B\frac{\partial^2 \lambda^b}{\partial x^m \partial K}\beta - \frac{B}{S}\frac{\partial v(\mathbf{X}^*)}{\partial K}. \end{aligned}$$

The determinant is $|H| = -\frac{1}{S}(1 - \lambda^b(0)\beta) - \frac{B}{S^2}\lambda^{b\prime}(0)\beta < 0$. This completes the proof of Proposition 3. ■

Proof of Proposition 4

As stated in the main text, for $\alpha S \geq B$ the intermediary can achieve the highest possible profit by choosing to be a pure middleman. What remains here is to prove the proposition for $\alpha S < B$. Applying the analysis in the previous section, we derive that the seller value equals, $W(x^s) = (1 - e^{-x^s} - x^s e^{-x^s})(1 - f)$ and the indifferent condition for buyers becomes, $V^m(x^m) = V^s(x^s)$ where $V^m(x^m) = \min\{\frac{K}{x^m}, 1\}(1 - p^m)$ and $V^s(x^s) = e^{-x^s}(1 - p^s)$. The binding participation constraint for buyers implies that $p^m = 1 - \frac{\lambda^b}{\min\{\frac{K}{x^m},1\}}$ and $f = 1 - \frac{\lambda^b}{e^{-x^s}}$, and the binding condition, (19), implies that $p^w = (1 - e^{-x^s} - x^s e^{-x^s})\frac{\lambda^b}{e^{-x^s}}$.

To guarantee $f \geq 0$, it is sufficient to assume that

$$\lambda^b \leq e^{-\frac{B-\alpha S}{S}}.$$

This also guarantees $p^m - p^w = 1 - \lambda^b(1 - \frac{1-e^{-x^s}-x^s e^{-x^s}}{e^{-x^s}}) > 0$ and that profits are non negative.

Using all these expressions of prices and fee, we can write the profit function as

$$\Pi(x^m, K) = (S-K)(1-e^{-x^s})(1 - \frac{\lambda^b}{e^{-x^s}}) + \min\{K, x^m\} - x^m\lambda^b - K(1 - e^{-x^s} - x^s e^{-x^s})\frac{\lambda^b}{e^{-x^s}},$$

where $x^s = \frac{B-x^m}{S-K}$. Differentiation yields

$$\frac{\partial \Pi(x^m, K)}{\partial x^m} = \frac{S}{S-K}\frac{1-e^{-x^s}}{e^{-x^s}}\lambda^b + \frac{\partial \min\{K, x^m\}}{\partial x^m} - e^{-x^s}, \tag{31}$$

which is positive if $\min\{K, x^m\} = x^m$. Hence, the solution has to satisfy $x^m \geq K$.

Observe that: $\lim_{x^m \to B} \Pi(x^m, K) = \Pi$ and $\lim_{x^m \to 0} \Pi(x^m, K) = \tilde{\Pi}(0)$, where $\Pi = \alpha S - B\lambda^b$ is the profit for the pure middleman mode and $\tilde{\Pi}(0) = S(1 - e^{-\frac{B}{S}})(1 - \frac{\lambda^b}{e^{-\frac{B}{S}}})$ is the profit for the pure market-maker mode. Hence, as before, we can find a profit-maximizing intermediation mode using the following Lagrangian:

$$\mathcal{L} = \Pi(x^m, K) + \mu_k(x^m - K) + \mu_b(B - x^m) + \mu_0 K + \mu_s(\alpha S - K).$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x^m} = \frac{\partial \Pi(x^m, K)}{\partial x^m} + \mu_k - \mu_b = 0, \tag{32}$$

$$\frac{\partial \mathcal{L}}{\partial K} = \frac{\partial \Pi(x^m, K)}{\partial K} - \mu_k + \mu_0 - \mu_s = 0, \tag{33}$$

where

$$\frac{\partial \Pi(x^m, K)}{\partial K} = e^{-x^s} + x^s e^{-x^s} - \frac{S}{S-K}\frac{1-e^{-x^s}}{e^{-x^s}}\lambda^b x^s.$$

Suppose $x^m = B$. Then, we must have $\mu_k = 0$ (since $B > \alpha S \geq K$) and so (32) implies we also must have $\frac{\partial \Pi(x^m,K)}{\partial x^m}\,|_{(x^m=B)} = \mu_b \geq 0$. However, $\frac{\partial \Pi(x^m,K)}{\partial x^m}\,|_{(x^m=B)} = -1 < 0$, a contradiction. Hence, the solution must satisfy $x^m < B$ (and $\mu_b = 0$), i.e., an active platform.

Summing up the two first order conditions with $\mu_b = 0$,

$$\begin{aligned}
\mu_s - \mu_0 &= \frac{\partial \Pi(x^m, K)}{\partial K} + \frac{\partial \Pi(x^m, K)}{\partial x^m} \\
&= x^s e^{-x^s} + (1 - x^s)\frac{S}{S-K}\frac{1-e^{-x^s}}{e^{-x^s}}\lambda^b \\
&= -x^s \frac{\partial \Pi(x^m, K)}{\partial x^m} + \frac{S}{S-K}\frac{1-e^{-x^s}}{e^{-x^s}}\lambda^b > 0
\end{aligned}$$

where the last inequality follows from (32) and $\mu_b = 0$ that implies $\frac{\partial \Pi(x^m,K)}{\partial x^m} = -\mu_k \leq 0$. This implies $\mu_s > 0$, i.e., the binding resource constraint (18), which implies $K = \alpha S$. This completes the proof of Proposition 4. ■

Proof of Proposition 5

In our endowment economy, the middleman's inventory purchase influences market tightness not only in the C market platform, but also in the D market. Given all sellers are in the D market, the probability that a buyer meets a seller available for trade in the D market changes from $\lambda^b e^{-x^s}$ to $\lambda^b \frac{S-K}{S} e^{-x^s}$. With this change and using the analysis of multi-market search shown in the previous section, the value of sellers, $W(x^s) = (1-e^{-x^s} - x^s e^{-x^s})(v(x^m, K) - f)$ and the middleman's price, $p^m = 1 - \lambda^b \frac{S-K}{S} e^{-x^s} - \frac{x^m e^{-x^s}}{\min\{x^m, K\}}(v(x^m, K) - f)$, where $v(x^m, K) = 1 - \lambda^b \frac{S-K}{S} e^{-x^s}$. Substituting these expressions into the profit function, it becomes immediate that the profit is strictly increasing in the fee f. Hence, the incentive constraints are binding, $f = v(x^m, K)$. Using this result, we can write the profit function as

$$\Pi(x^m, K) = (S-K)(1-e^{-x^s})(1 - \lambda^b \frac{S-K}{S} e^{-x^s}) + \min\{K, x^m\}(1 - \lambda^b \frac{S-K}{S} e^{-x^s}),$$

where $x^s = \frac{B-x^m}{S-K}$. Differentiation yields

$$\begin{aligned} \frac{\partial \Pi(x^m, K)}{\partial x^m} &= -e^{-x^s}\left(1 - \lambda^b \frac{S-K}{S} e^{-x^s} - \lambda^b \frac{S-K}{S}(1-e^{-x^s})\right) - \frac{\min\{K, x^m\}}{S}\lambda^b e^{-x^s} \\ &\quad + \frac{\partial \min\{K, x^m\}}{\partial x^m}\left(1 - \lambda^b \frac{S-K}{S} e^{-x^s}\right), \end{aligned}$$

which is negative if $\min\{K, x^m\} = K$. Hence, the solution has to satisfy $x^m \le K$.

Suppose $x^m = B$. Then,

$$\frac{\partial \Pi(x^m, K)}{\partial x^m} |_{x^m=B} = -\frac{B}{S}\lambda^b < 0.$$

Hence, the solution has to be $x^m < B$, i.e., an active platform.

The Lagrangian the becomes,

$$\mathcal{L} = \Pi(x^m, K) + \mu_0 x^m + \mu_k (K - x^m) + \mu_s(\alpha S - K).$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x^m} = \frac{\partial \Pi(x^m, K)}{\partial x^m} + \mu_0 - \mu_k = 0, \quad (34)$$

$$\frac{\partial \mathcal{L}}{\partial K} = \frac{\partial \Pi(x^m, K)}{\partial K} + \mu_k - \mu_s = 0, \quad (35)$$

where

$$\begin{aligned} \frac{\partial \Pi(x^m, K)}{\partial K} &= x^s e^{-x^s}\left(1 - \lambda^b \frac{S-K}{S} e^{-x^s} + x^s \lambda^b \frac{S-K}{S}(1-e^{-x^s})\right) + \frac{x^s x^m}{S}\lambda^b e^{-x^s} \\ &\quad + \left(1-e^{-x^s}\right)\left(1 - 2\lambda^b \frac{S-K}{S} e^{-x^s}\right) + \frac{x^m}{S}\lambda^b e^{-x^s}. \end{aligned}$$

Combining (34) and (35),

$$\frac{\partial \Pi(x^m, K)}{\partial x^m} + \frac{\partial \Pi(x^m, K)}{\partial K} = x^s e^{-x^s}\left[\left(1 - \lambda^b \frac{S-K}{S} e^{-x^s}\right) + \frac{S-K}{S}(1-e^{-x^s})\lambda^b + \frac{x^m}{S}\lambda^b\right] = \mu_s - \mu_0,$$

which implies $\mu_s > 0$ and $K = \alpha S$. This completes the proof of Proposition 5. ■

Proof of Proposition 6

⊙ Single-market search. Note $K = x^m$ in equilibrium. With single-market search, the problem of a middleman can be described as

$$\tilde{\Pi}(K) = \max_{p^m, K} \min\{K, B\} p^m - C(K) \text{ s.t. } V^m = 1 - p^m \ge \lambda^b.$$

The solution is $p^m = 1 - \lambda^b$ and $K = B$ (as $C'(K) < 1 - \lambda^b$ for $K < B$), and so the middleman's profit is $\tilde{\Pi}(B) = B(1 - \lambda^b) - C(B)$. When a platform is open, individual sellers solve:

$$\max_x (1 - e^{-x})(p - f^s) \;\; \text{s.t. } V^s = \frac{1 - e^{-x}}{x}(1 - p - f^b)$$

The optimal solution is described as $V^s = e^{-x}(1 - f)$ with $f \equiv f^b + f^s$. In an equilibrium with a market-making middleman, it has to hold that $x = x^s$ and $V^s = V^m = 1 - p^m \geq \lambda^b$, which implies $p^m = 1 - \lambda^b$ and $f = 1 - \lambda^b e^{x^s}$. Hence, the problem of a market-making middleman is given by (20), with $\lim_{x^m \to B} \Pi(x^m) = \tilde{\Pi}(B)$. Observe in (21) that: $\Theta_{Sfoc}(B) = -C'(B) < 0$ and $\Theta'_{Sfoc}(x^m) = -\frac{1}{S}(e^{-x^s} + \lambda^b e^{x^s}) - C''(x^m) < 0$. Therefore, a market-making middleman is profit-maximizing if

$$\Theta_{Sfoc}(0) = 1 - e^{-\frac{B}{S}} + \lambda^b (e^{\frac{B}{S}} - 1) - C'(0) > 0. \tag{36}$$

Otherwise, the pure market-maker mode is selected.

⊙ Multi-market search. Note $K = x^m$ in equilibrium. With multi-market search, the only modification is to introduce the marginal inventory cost $C'(x^m)$ in the first order condition. Using (22), we can write the first order condition as:

$$\Theta_{Mfoc}(x^m) \equiv (1 - e^{-x^s})(1 - 2\lambda^b e^{-x^s}) - \frac{\lambda^b x^m e^{-x^s}}{S} - C'(x^m) = 0.$$

Observe that: $\Theta_{Mfoc}(B) = -\frac{\lambda^b B}{S} - C'(B) < 0$ and

$$\Theta'_{Mfoc}(x^m) = -\frac{e^{-x^s}}{S}\left[1 - \lambda^b e^{-x^s} + 3\lambda^b(1 - e^{-x^s})\right] - \frac{\lambda^b x^m e^{-x^s}}{S^2} - C''(x^m) < 0.$$

Therefore, a market-making middleman is profit-maximizing if

$$\Theta_{Mfoc}(0) = (1 - e^{-\frac{B}{S}})(1 - 2\lambda^b e^{-\frac{B}{S}}) - C'(0) > 0. \tag{37}$$

Otherwise, a pure market-maker is selected.

⊙ Comparison. Comparing (36) and (37),

$$\Theta_{Mfoc}(0) \quad = \quad \Theta_{Sfoc}(0) - \lambda^b \left[2e^{-x^s}(1 - e^{-x^s}) + e^{x^s} - 1\right] < \Theta_{Sfoc}(0),$$

implying that whenever a middleman sector is active, i.e., $x^m > 0$, with multi-market search, it has to be active with single-market search as well. Further, whenever a market-making middleman mode is selected, since $\Theta'_{Mfoc}(x^m) = \frac{\partial^2 \Pi(x^m)}{\partial x^{m2}} < 0$, the comparison given in the main text implies that the platform sector x^s $(= \frac{B - x^m}{S})$ is always larger with multi-market search than single-market search. This completes the proof of Proposition 6. ■

Proof of Proposition 8

⊙ Single-market search. First of all, the expected value of buyers to search E, V_E, is given as follows. If E is a middleman with a price p_E, then $V_E = 1 - p_E$. If E is a market-maker with fees f_E, then $V_E = e^{-\frac{B}{S}}(1 - f_E)$ under unfavorable beliefs towards I and $V_E = 0$ under favorable beliefs towards I.

If I is a pure middleman, then it makes a profit of Bp^m with price $p^m = 1 - V_E$. If I activates a platform, it must satisfy the participation constraints,

$$\eta^s(x^s)(1 - p^s - f^b) \geq V_E,$$
$$p^s - f^s \geq 0.$$

Under these conditions, it holds that

$$f \equiv f^s + f^b \leq 1 - V_E.$$

Hence, the resulting profit of I satisfies $S(1-e^{-x^s})f + x^m p^m < (Sx^s + x^m)\max\{f, p^m\} \le B(1-V_E)$. That is, the pure middleman mode dominates any other modes with an active platform. Therefore, under single-market search, I must be a pure middleman in all possible equilibria.

⊙ Multi-market search: E is a pure middleman. With multi-market search, when E is a pure middleman, an active platform of I has to satisfy the incentive constraints,

$$\begin{aligned} 1-p^s-f^b &\ge 1-p_E \\ p^s-f^s &\ge 0. \end{aligned}$$

These constraints imply: $f \le p_E$. Similarly, an active middleman sector has to satisfy $p^m \le p_E$. Then, if $\max\{p^m, f\} \le p_E$, then I can be a market-making middleman, and if

$$\min\{p^m, f\} \le p_E,$$

then trade can occur in either one of the sectors, and so I can be an active intermediary. The profit of I is

$$S(1-e^{-x^s})f + x^m p^m.$$

Noting $x^s = \frac{B-x^m}{S}$, we see from this expression that the profit maximization requires that $x^m = B$ with $p^m = f = p_E$. Hence an active platform is not profitable. Then, since the two intermediaries compete with price, any equilibrium must be subject to the Bertrand undercutting, leading to $p^m = p_E = 0$ and zero profits.

⊙ Multi-market search: E is a market-maker. When E is a pure market-maker, the expected value of buyers to search E is modified to

$$V_E = \lambda^b e^{-x^s}(1-f_E),$$

which is essentially the same as in the baseline setup (i.e., we assume, to be line with the baseline setup, the structure of a marketplace E offers is the same as that of our D market), except that f_E is subtracted from the total surplus, and that, to ease the exposition, we set $\beta = 1$. With this modification, as long as

$$\max\{p^m, f\} \le 1 - V_E,$$

trade can happen in at least one of the sectors and so I can be an active intermediary.

We now show that an active platform is used in equilibrium with the following steps. In Step 1, we show that a pure strategy equilibrium does not exist in the game between I, who selects $(f, p^m) \in [0, \bar{f}] \times [0, \bar{p}]$ with a payoff $\Pi_I = \Pi_I(f, p^m \mid f_E)$, and E, who selects $f_E \in [0,1]$ with a payoff $\Pi_E = \Pi_E(f_E \mid f, p^m)$. Here, we set $\bar{f}, \bar{p} > 1$ and $f > 1$ ($\bar{p} > 1$) leads to an inactive platform (middleman sector). In Step 2, we apply Theorem 5 of Dasgupta and Maskin (1986) and show that because Π_I (Π_E) is bounded and weakly lower semi-continuous in f and p^m (in f_E), and $\Pi_I + \Pi_E$ is upper semi-continuous, there exists a mixed strategy equilibrium. Given $f_E \in [0,1)$, which is selected with positive probability, this is the same situation as in the benchmark with exogenous D market, as shown in Proposition 2. Hence, this results implies that I activates a platform with positive probability in equilibrium.

Step 1 When E is a market-maker with multiple-market search, there exists no pure strategy equilibrium in the game between I and E.

Proof of Step 1. Suppose that first E chooses $f_E > 0$. The best response of I involves $\min\{f, p^m\} = 1 - V_E$, where $V_E = V_E(f_E)$ is determined above. However, by lowering the fee to $f_E - \epsilon$, with an arbitrary small $\epsilon > 0$, E will experience a discrete jump in demand, leading to $\Pi_E(f_E \mid \cdot) = (B - x^m - S(1-e^{-x^s}))\lambda^b f_E < B\lambda^b(f_E - \epsilon) = \Pi_E(f_E - \epsilon \mid \cdot)$. Hence, any $f_E > 0$ cannot be part of equilibrium. Suppose next that E chooses $f_E = 0$ (leading to $\Pi_E(0 \mid \cdot) = 0$). As shown in Proposition 2, I will activate a platform, and so $x^m < B$. This implies that by setting $f_E > 0$, E can make a positive profit

$\Pi_E(f_E \mid \cdot) = (B - x^m - S(1-e^{-x^s}))\lambda^b f_E > 0$. Hence, $f_E = 0$ cannot be an equilibrium. Therefore, there is no pure strategy equilibrium.

Step 2 When E is a market-maker with multiple-market search, there exists a mixed strategy equilibrium in the game between I and E.

Proof of Step 2. Given Theorem 5 of Dasgupta and Maskin (1986), it is sufficient to show that Π_I (Π_E) is bounded and weakly lower semi-continuous in f and p^m (in f_E), and $\Pi_I + \Pi_E$ is upper semi-continuous. Clearly, Π_I (Π_E) is bounded in $(f, p^m) \in [0, \bar{f}] \times [0, \bar{p}]$ (in $f_E \in [0, 1]$).

Both of the profit functions are continuous except at

$$\min\{f, p^m\} = 1 - V_E(f_E), \tag{38}$$

so we shall pay attention to this discontinuity point.

First, we show that $\Pi_I(f, p^m \mid f_E)$ is weakly lower semi-continuous in (f, p^m). Give the discontinuous point in (38), we have

$$\Pi_I(f, p^m \mid f_E) = \begin{cases} S(1-e^{-x^s})f + x^m p^m, & \text{if } \min\{f, p^m\} \le 1 - V_E(f_E) \\ 0 & \text{otherwise,} \end{cases}$$

where in the second situation, the price/fee of I are not comparable to the fee of E, hence agents will trade via E, rather than I, and so I will become inactive. Consider some $f_\varepsilon \in [0, 1]$, and some $f, p^m > 0$ such that $\min\{f, p^m\} = 1 - V_E(f_E)$. For any sequence $\{(f^{(j)}, p^{m(j)})\}$ converging to (f, p^m) such that no two $f^{(j)}$'s, and no two $p^{m(j)}$'s are the same, and $f^{(j)} \le f, p^{m(j)} \le p^m$, we must have $\min\{f^{(j)}, p^{m(j)}\} \le 1 - V_E(f_E)$. Hence,

$$\lim_{j\to\infty} \Pi_I(f^{(j)}, p^{m(j)} \mid f_E) = \Pi_I(f, p^m \mid f_E),$$

satisfying the definition of weakly lower semi-continuity (see Definition 6 in page 13 of Dasgupta and Maskin, 1986, or condition (9) in page 384 of Maskin, 1986).

Second, we shall show that $\Pi_E(f_E \mid f, p^m)$ is lower semi-continuous in f_E. Consider a potential discontinuity point $f_0 \in (0, 1)$ satisfying (38) such that

$$\Pi_E(f_E \mid f, p^m) = \begin{cases} B\lambda^b f_E, & \text{if } f_E < f_0 \\ (B - x^m - S(1-e^{-x^s}))\lambda^b f_E & \text{if } f_E \ge f_0. \end{cases}$$

Clearly, this function is lower semi-continuous, since for every $\epsilon > 0$ there exists a neighborhood U of f_0 such that $\Pi_E(f_E \mid \cdot) \ge \Pi_E(f_0 \mid \cdot) - \epsilon$ for all $f_E \in U$.

Finally, we prove the upper semi-continuity of $\Pi_I + \Pi_E$. For this purpose, consider all sequences of $\{f^{(j)}, p^{m(j)}, f_E^{(j)}\}$ that converges to $\{\hat{f}, \hat{p}^m, \hat{f}_E\}$ that satisfies $\min\{\hat{f}, \hat{p}^m\} = 1 - V_E(\hat{f}_E)$.

Consider first an extreme in which case $\min\{f^{(j)}, p^{m(j)}\} \le 1 - V_E(f_E^{(j)})$ for all j. As the equilibrium is where I is visited prior to E, we must have

$$\lim_{j\to\infty} \Pi_I(f^{(j)}, p^{m(j)} \mid f_E^{(j)}) + \Pi_E(f_E^{(j)} \mid f^{(j)}, p^{m(j)}) = \Pi_I(\hat{f}, \hat{p}^m \mid \hat{f}_E) + \Pi_E(\hat{f}_E \mid \hat{f}, \hat{p}^m).$$

Consider next the other extreme in which $\min\{f^{(j)}, p^{m(j)}\} > 1 - V_E(f_E^{(j)})$ for all j. Then, the equilibrium is where E is visited prior to I, and we must have

$$\lim_{j\to\infty} \Pi_I(f^{(j)}, p^{m(j)} \mid f_E^{(j)}) + \Pi_E(f_E^{(j)} \mid f^{(j)}, p^{m(j)}) = B\lambda^b \hat{f}_E.$$

If $\hat{f} \ge \hat{p^m}$, then

$$\Pi_I(\hat{f}, \hat{p}^m \mid \hat{f}_E) + \Pi_E(\hat{f}_E \mid \hat{f}, \hat{p}^m) = B\hat{p}^m = B(1 - \lambda^b(1 - \hat{f}_E)) > B\lambda^b \hat{f}_E.$$

If $\hat{f} < \hat{p^m}$, then

$$\Pi_I(\hat{f}, \hat{p}^m \mid \hat{f}_E) + \Pi_E(\hat{f}_E \mid \hat{f}, \hat{p}^m) = B(1 - e^{-\frac{B}{S}})\hat{f} + B\lambda^b e^{-\frac{B}{S}} \hat{f}_E > B[(1 - e^{-\frac{B}{S}}) + \lambda^b e^{-\frac{B}{S}}]\hat{f}_E > B\lambda^b \hat{f}_E.$$

Thus,

$$\lim_{j\to\infty} \Pi_I(f^{(j)}, p^{m(j)} \mid f_E^{(j)}) + \Pi_E(f_E^{(j)} \mid f^{(j)}, p^{m(j)}) < \Pi_I(\hat{f}, \hat{p}^m \mid \hat{f}_E) + \Pi_E(\hat{f}_E \mid \hat{f}, \hat{p}^m).$$

As these two extreme cases give the upper and lower bounds respectively, all the other sequences give some limits in between. Therefore,

$$\lim_{j\to\infty} \Pi_I(f^{(j)}, p^{m(j)} \mid f_E^{(j)}) + \Pi_E(f_E^{(j)} \mid f^{(j)}, p^{m(j)}) \le \Pi_I(\hat{f}, \hat{p}^m \mid \hat{f}_E) + \Pi_E(\hat{f}_E \mid \hat{f}, \hat{p}^m),$$

for any of the sequences converging to $\{\hat{f}, \hat{p}^m, \hat{f}_E\}$, and so $\Pi_I + \Pi_E$ is upper semi-continuous. This completes the proof of Proposition 8. ■

Web Appendix: Additional proofs and details of the empirical implementation

Participation fees

In this Additional Appendix, we show that our main result does not change in a version of our model where the middleman's supply is not observable in the participation stage, but instead the intermediary can use participation fees/subsidy. Suppose now that in the first stage the intermediary announces a set of fees $F \equiv \{f^b, f^s, g^b, g^s\}$ for the platform, where $f^b,\ f^s \in [0,1]$ is a transaction fee charged to a buyer or a seller, respectively, and $g^b,\ g^s \in [-1,1]$ is a registration fee charged to a buyer or a seller, respectively.

As is consistent with the main analysis, we follow the literature of two-sided markets and assume that agents hold pessimistic beliefs on the participation decision of agents on the other side of the market (Caillaud and Jullien, 2003). Agents believe that the intermediary would never supply anything at all unless the C market attracts some buyers. This is the worst situation for the intermediary, and (**??**) is not the right participation constraint. A pessimistic belief of sellers means that sellers believe the number of buyers participating in the C market is zero whenever

$$\lambda^b \beta > -g^b,$$

where $\lambda^b \beta$ is the expected payoff of buyers in the D market and $-g^b$ is the payoff buyers receive in the C market (it is a participation subsidy when $g^b < 0$).

Single-market search: To induce the participation of agents under those beliefs, the best the intermediary can do is to use a divide-and-conquer strategy, denoted by h. To divide buyers and conquer sellers, referred to as $h = D_b C_s$, it is required that

$$D_b \quad : \quad -g^b \geq \lambda^b \beta, \tag{39}$$
$$C_s \quad : \quad W - g^s \geq 0. \tag{40}$$

The divide-condition D_b tells us that the intermediary should subsidize the participating buyers so that they receive at least what they would get in the D market, *even if the C market is empty.* This makes sure buyers will participate in the C market whatever happens to the other side of the market. The conquer-condition C_s guarantees the participation of sellers, by giving them a nonnegative payoff – the participation fee $g^s \geq 0$ should be no greater than the expected value of sellers in the C market, $W = W(x^s)$. Observing that the intermediary offers buyers enough to participate, sellers understand that all buyers are in the C market, the D market is empty, and so the expected payoff from the D market is zero. Here, the expected value of sellers in the C market W is defined under the sellers' belief that the intermediary will select the capacity level optimally given the full participation of buyers.

Similarly, a strategy to divide sellers and conquer buyers, referred to as $h = D_s C_b$, requires that

$$D_s \quad : \quad -g^s \geq \lambda^s (1-\beta), \tag{41}$$
$$C_b \quad : \quad V - g^b \geq 0. \tag{42}$$

where $V = \max\{V^s(x^s), V^m(x^m)\}$ is the expected value of buyers in the C market.

Given the participation decision of agents described above, the intermediary's problem of determining the intermediation fees $F = \{f^b, f^s, g^b, g^s\}$ for $h = \{D_b C_s, D_s C_b\}$ is described as

$$\Pi = \max_{F,h} \{B g^b + S g^s + \max_{p^m, K} \Pi(p^m, f, K)\},$$

subject to (39) and (40) if $h = D_b C_s$, or (41) and (42) if $h = D_s C_b$. Here, $B g^b$ and $S g^s$ are participation fees from buyers and sellers, respectively, and $\Pi(\cdot)$ is the expected profit in the C market described above. Under either of the divide-and-conquer strategies, the choice of participation fees g^i, $i = b, s$, does not influence anyone's behaviors in the C market. The choice of transaction fees affects the expected value of agents and thus the participation fees and intermediary's profits. However, it does not alter the original solution, a pure middleman, remains optimal.

Proposition 9 *With unobservable capacity and with participation fees, the intermediary sets* $f > 1$, $p^m = 1$ *and* $K = B$*. All the buyers buy from the middleman,* $x^m = B$*, and the platform is inactive,* $x^s = 0$*. The intermediary makes profits,*

$$\Pi = B - \min\{B\lambda^b\beta, S\lambda^s(1-\beta)\},$$

guaranteeing the participation of agents by $h = D_bC_s$ *if* $\beta < \frac{1}{2}$ *and* $h = D_sC_b$ *if* $\beta > \frac{1}{2}$.

Proof. Consider first $h = D_bC_s$. Then, by (39) and (40), $g^b = -\lambda^b\beta$ and $g^s = W$. For $f > 1$, no buyers go to the platform $x^s = 0$ and all buyers are in the middleman sector $x^m = B$, yielding $g^s = W = 0$. By selecting $K = B$ and $p^m = 1$, the intermediary makes profits,

$$\Pi = -B\lambda^b\beta + \Pi(p^m, 1, B) = (-\lambda^b\beta + 1)B.$$

To show that this is indeed the maximum profit, we have to check two possible cases. Suppose $f = f^b + f^s \leq 1$ and $K = 0$. Then, $x^s = \frac{B}{S}$ and $x^m = 0$, and $g^s = W(B/S) \geq 0$, if there is a non-negative surplus in the platform for buyers, $f^b + p^s \leq 1$, and for sellers, $f^s \leq p^s$. The resulting profit satisfies

$$\begin{aligned} Bg^b + Sg^s + \Pi(p^m, f, 0) &= -B\lambda^b\beta + S(1-e^{-\frac{B}{S}})(p^s - f^s) + S(1-e^{-\frac{B}{S}})f \\ &= -B\lambda^b\beta + S(1-e^{-\frac{B}{S}})(f^b + p^s) \\ &< -B\lambda^b\beta + B = \Pi \end{aligned}$$

for all $f^b + p^s \leq 1$. Hence, this is not profitable.

Suppose $f = f^b + f^s \leq 1$ and $K \in (0, B]$, and both sectors have a non-negative surplus to buyers, i.e., $p^m \leq 1$ and $f^b + p^s \leq 1$. This leads to $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$ that satisfy the add-up requirement (1) and the indifferent condition (2). Then, $g^s = W(x^s) \geq 0$, and the resulting profit is

$$\begin{aligned} &Bg^b + Sg^s + \Pi(p^m, f, K) \\ &\quad = -B\lambda^b\beta + S(1-e^{-x^s})(p^s - f^s) + S(1-e^{-x^s})f + \min\{K, x^m\}p^m \\ &\quad < -B\lambda^b\beta + Sx^s(f^b + p^s) + x^m p^m \\ &\quad \leq -B\lambda^b\beta + (Sx^s + x^m)\max\{f^b + p^s, p^m\} \\ &\quad \leq -B\lambda^b\beta + B = \Pi \end{aligned}$$

for all $f^b + p^s \leq 1$ and $p^m \leq 1$. Hence, this is not profitable either. All in all, no deviation is profitable for $h = D_bC_s$.

Consider next $h = D_sC_b$. Then, by (41) and (42), $g^s = -\lambda^s(1-\beta)$ and $g^b = V$. When $f > 1$, no one go to the platform $x^s = 0$ and all buyers are in the middleman sector $x^m = B$ as long as $p^m \leq 1$. This yields $g^b = V = V^m(B) \geq 0$ and $\Pi(p^m, f, B) = Bp^m$ with $K = B$. The profits are

$$\Pi = -S\lambda^s(1-\beta) + B(1-p^m) + \Pi(p^m, f, K) = -S\lambda^s(1-\beta) + B.$$

To show that this is indeed the maximum profit, we have to check two possible cases. Suppose $f = f^b + f^s \leq 1$ and $K = 0$. Then, $x^s = \frac{B}{S}$ and $x^m = 0$, and $g^b = V = V^s(B/S) \geq 0$, if there is a non-negative surplus in the platform for buyers, $f^b + p^s \leq 1$, and for sellers, $f^s \leq p^s$. This leads to

$$\begin{aligned} Sg^s + Bg^b + \Pi(p^m, f, 0) &= -S\lambda^s(1-\beta) + B\frac{1-e^{-\frac{B}{S}}}{\frac{B}{S}}(1 - p^s - f^b) + S(1-e^{-\frac{B}{S}})f \\ &= -S\lambda^s(1-\beta) + S(1-e^{-\frac{B}{S}})(1 - p^s + f^s) \\ &< -S\lambda^s(1-\beta) + B = \Pi \end{aligned}$$

for all $f^s \leq p^s$. Hence, this is not profitable.

Suppose $f = f^b + f^s \leq 1$ and $K \in (0, B]$, and both sectors have a non-negative surplus to buyers, i.e., $p^m \leq 1$ and $f^b + p^s \leq 1$. This leads to $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$ that satisfy the add-up constraint (1),

$Sx^s + x^m = B$, and the indifferent condition (2), $V^s(x^s) = V^m(x^m)$. Then, $g^b = V = V^s(x^s)$, and the resulting profit is

$$
\begin{aligned}
&Sg^s + Bg^b + \Pi(p^m, f, K) \\
&= -S\lambda^s(1-\beta) + B\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + \min\{K, x^m\}p^m.
\end{aligned}
$$

There are two cases. Suppose $K \geq x^m$. Then, the indifferent condition (2) implies that

$$
p^m = 1 - \frac{1-e^{-x^s}}{x^s}(1-p^s-f^b).
$$

Applying this expression to the profits, we get

$$
\begin{aligned}
&Sg^s + Bg^b + \Pi(p^m, f, K) \\
&= -S\lambda^s(1-\beta) + B\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + x^m\left(1 - \frac{1-e^{-x^s}}{x^s}(1-p^s-f^b)\right) \\
&= -S\lambda^s(1-\beta) + (B - x^m)\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + x^m \\
&= -S\lambda^s(1-\beta) + S(1-e^{-x^s})(1-p^s+f^s) + x^m \\
&< -S\lambda^s(1-\beta) + B
\end{aligned}
$$

for all $f^s \leq p^s$. Suppose $K < x^m$. Then, the indifferent condition implies that

$$
p^m = 1 - \frac{x^m}{K}\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b).
$$

Applying this expression to the profits, we get

$$
\begin{aligned}
&Sg^s + Bg^b + \Pi(p^m, f, K) \\
&= -S\lambda^s(1-\beta) + B\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + K\left(1 - \frac{x^m}{K}\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b)\right) \\
&= -S\lambda^s(1-\beta) + (B - x^m)\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + K \\
&= -S\lambda^s(1-\beta) + S(1-e^{-x^s})(1-p^s+f^s) + K \\
&< -S\lambda^s(1-\beta) + B
\end{aligned}
$$

for all $f^s \leq p^s$. Hence, any deviation is not profitable for $h = D_sC_b$.

Finally, since the intermediary makes the maximum revenue B for either h, which side should be subsidized is determined by the required costs: noting $B\lambda^b = S\lambda^s$, we have $B\lambda^b\beta \gtreqless S\lambda^s(1-\beta) \iff \beta \gtreqless \frac{1}{2}$. This completes the proof of Proposition 9. ■

Multi-market search: With multiple-market search, any non-positive registration fee can ensure that agents are in the C market, since the participation to the C market is not exclusive. Hence, attracting one side of the market becomes less costly. By contrast, conquering the other side becomes more costly, since the conquered side still holds the trading opportunity in the D market. The D_sC_b condition with multiple-market search is

$$
\begin{aligned}
D_s &: \ -g^s \geq 0, \\
C_b &: \ \max\{V^s(x^s), V^m(x^m)\} - g^b \geq \lambda^b e^{-x^s}\beta(1-c).
\end{aligned}
$$

The divide-condition D_s tells that now a non-positive fee is sufficient to convince one side to participate. The conquer-condition C_b now needs to compensate for the outside option in the D market. Similarly, the D_bC_s condition becomes

$$
\begin{aligned}
D_b &: \ -g^b \geq 0, \\
C_s &: \ W(x^s) - g^s \geq \lambda^s\xi(x^s, x^m)(1-\beta)(1-c).
\end{aligned}
$$

Participation fees are designed to induce buyers and sellers' participation. Once agents join the C market, the participation fees become sunk costs, and will not influence their trading decision.

The intermediary's problem of choosing $F = \left\{f^b, f^s, g^b, g^s\right\}$ together with $h = \{D_bC_s, D_sC_b\}$ and $p^m, K \in [0, B]$ are described as

$$\Pi = \max_{F,h,K} \left\{ Bg^b + Sg^s + \max_{p^m} \Pi(p^m, f, K) \right\}, \tag{43}$$

where $\Pi(p^m, f, K) = S(1 - e^{-x^s})f + \min\{K, x^m\} p^m - Kc$. Besides the divide-and-conquer constraints, this maximization problem is also subject to the incentive constraints as described in the main text.

Proposition 10 *In the extended problem described in (43) with unobservable capacity, participation fees and multiple-market search, the determination of the profit-maximizing intermediation mode is identical to the one described in Proposition 2, with $g^i = 0$, $i = s, b$.*

Proof. It suffices to prove that the solution is $g^i = 0$, $i = s, b$ for each intermediation mode, since then the problem (43) will become identical to the one we have already solved in the main text. For a pure middleman mode ($x^m = B$), the intermediary sets $g^b = 0$ to divide buyers, with $p^m = 1 - \lambda^b\beta(1 - c)$ satisfying (7). For a pure market-maker mode ($x^s = 0$), either with D_bC_s or D_sC_b, the intermediary sets the transaction fee to satisfy the binding incentive constraint (13), $f = v(0, 0) = \left[1 - \lambda^b e^{-B/S} - \lambda^s\xi(0, 0)\right](1 - c)$, and $g^b = g^s = 0$.

For a hybrid mode, the intermediary's problem is subject to the incentive constraint (13), and p^m satisfying (16) so that buyers are indifferent between the two modes. We can rewrite the maximization problem (43) as a two-stage problem over a vector $\mathbf{X} \equiv (x^m, f, K) \in \mathbb{X}$, where $\mathbb{X} \equiv [0, B] \times [0, 1] \times [0, K]$:

$$\begin{aligned} &\text{Stage 1: } \max_{(f,K)} Bg^b(\mathbf{X}) + Sg^s(\mathbf{X}) + \Pi(x^m(f, K), f, K) \\ &\qquad s.t.\ 0 \le f \le v(x^m(f, K), K),\ 0 \le K \le B. \\ &\text{Stage 2: } \max_{x^m} \Pi(x^m, f, K) \\ &\qquad s.t.\ f \le v(x^m, K),\ 0 \le x^m \le B, \end{aligned} \tag{$\mathcal{A}$}$$

where $g^b(\mathbf{X})$ and $g^s(\mathbf{X})$ are given by the binding divide-and-conquer conditions,

$$g^b(\mathbf{X}) = 0,\ g^s(\mathbf{X}) = \left(1 - e^{-x^s} - x^s e^{-x^s}\right)(v(x^m, K) - f),$$

if $h = D_bC_s$, or

$$g^s(\mathbf{X}) = 0,\ g^b(\mathbf{X}) = e^{-x^s}(v(x^m, K) - f).$$

if $h = D_sC_b$. As our objective is to prove $g^i(\mathbf{X}) = 0$, $i = s, b$, all that remains here is to show that $f = v(x^m, K)$ at the solution. However, it is immediate that the objective function in ($\mathcal{A}$) is strictly increasing in f and any change in f ($< v(x^m, K)$) does not influence the other constraints. Hence, as in the original problem, we must have $f = v(x^m, K)$. This completes the proof of Proposition 10. ∎

Empirical Appendix

Data and Variables. From *www.amazon.com* and *www.ebay.com*, we retrieve all paginated results listed in the category of Amazon called: *"All Pans"*, which is a subcategory of *"Home & Kitchen: Kitchen & Dining: Cookware"*. This subcategory includes 400 pages of more than 9000 products as of August 2018.[22] For each pan, we obtain the price (*price*), the sales rank in the category *"Home & Kitchen"* (*rank*), the listing days since the first listed date on Amazon by either Amazon or some other sellers on Amazon's market-maker platform (*listedDays*), the number of third-party sellers that sell this product on

[22] The URL of the list of all pans is *https://www.amazon.com/pans/b?node=3737221* (visited on August 24, 2018).

Amazon (*sellersAmazon*), whether the product is sold by Amazon itself (*sellByAmazon*) and the title of the product.

Sellers could offer various prices for a product on Amazon. We obtain price information from the default page Amazon displays when users search for a product. This gives us the price at which the majority of transactions are processed. Amazon does not publish sales data but does provide a sales ranking for each product. Since ranking information is provided at different levels of categories, in order to make the sales ranking as comparable as possible, we adopt the ranking at the highest possible level *"Home & Kitchen"*. This gives us the variable *rank*.

The title of the product is used to link each product on Amazon to the outside option available at eBay as the theory develops. For each product collected on Amazon, we search its "Amazon product name" on eBay to obtain all related offers. As a proxy for the buyers' matching probability in the decentralized market λ^b, we count the number of all the offers shown in eBay's raw search result. We call this variable *sellersEbayAll*. Admittedly, this is a very noisy measure. EBay tends to provide a long list with offers that are only loosely related to the product. For example, in some cases a pan offered on eBay only matches with some key features of a pan offered on Amazon such as size but it does not match other features such as materials. In this case, we compare the similarity between the eBay product title and the Amazon product title. In some other cases, the titles are similar but the products turn out to be different. For example, searching a pan on eBay only yields an offer of the lid of the same pan on Amazon. To solve this issue, we use the following two-step procedure. We first select offers with product names similar to the Amazon product name.[23] We then refine the list by restricting the offer price between 0.5 and 1.5 times the Amazon price. The rationale for this procedure is that if the offer price is far away from the Amazon offer, the product is likely to vary in quality or could even be a distinct product. Counting the number of sellers in this refined list leads to another proxy for λ^b, *sellersEbayRefined*. This is a more precise measure for the relevant number of sellers on eBay. We will use *sellersEbayRefined* in our main regressions, and use *sellersEbayAll* as a robustness check.

As an alternative proxy for λ^b, we could use the number of sellers on eBay relative to that of Amazon,[24]

$$sellersEbayRelative = \frac{sellersEbayRefined}{sellersAmazon}.$$

This measure proxies the relative success probability of meeting a seller on eBay versus Amazon. It is constructed based on a typical buyer's online shopping experience. When a buyer discovers dozens of sellers on Amazon, it is relatively less likely that he can find even better offers outside Amazon, so the perceived outside value of going to eBay is low. In contrast, for a buyer who observes only few sellers on Amazon, the expected payoff of searching on the outside market is high. *sellersEbayRelative* is therefore likely to be positively correlated with λ^b.

As a proxy for buyers' bargaining power in the outside market, β, we compute the price difference between eBay and Amazon. For each product, we find the median price in the refined eBay offer list and compute the log of this price minus the log of Amazon price. This defines the variable *priceDiff*. We expect this variable to be negatively correlated with β (recall that a higher β implies a larger share of the surplus for the buyer so a lower price in the decentralized market).

Descriptive Analysis. We collected information for 9066 products on Amazon and found matched eBay offers for 7944 of them. Variables may have missing values leading to a smaller sample size. For example, ranking information might be provided not in the aggregate category *"Home & Kitchen"* but in some subcategories with incomplete ranking. We did try different (sub)categories to extract ranking information, and it turned out that *"Home & Kitchen"* gave us the largest valid sample with 7942 observations. In the regressions below, we exclude products without any matched eBay offers to avoid missing *priceDiff*, and exclude products without any third-party sellers on Amazon to avoid missing *sellersEbayRelative*. Finally, we only collect offers for brand new products.

[23] Here, we use the Fuzzy String Matching Library in Python which computes a score between 0 and 100, with 100 indicating the exact matching. The function `fuzz.token_set_ratio()` computes the score and only selects offers with a score higher than 80. We also tried other criteria scores such as 60 and 90. The results are robust.

[24] We use the number of third-party sellers (that is excluding Amazon if Amazon sells) in the denominator.

Table 2: Summary Statistics

Variables	Obs.	Mean	Std.	Min.	Max.
sellByAmazon	9066	0.32	0.46	0.00	1.00
listedDays	8168	1759.54	1458.12	8.00	6864.00
price	8856	64.03	107.00	0.01	2118.83
rank	7942	440711	314288	28	2581111
sellersAmazon	9066	3.70	5.36	0.00	77.00
sellersEbayAll	9066	14.69	14.13	0.00	60.00
sellersEbayRefined	9066	6.53	8.05	0.00	43.00
sellersEbayRelative	8487	3.30	5.35	0.00	43.00
priceDiff	7944	0.07	0.74	-5.08	6.86
sellersEbayAll_60	9066	20.88	15.87	0.00	62.00
sellersEbayRefined_60	9066	10.54	10.53	0.00	48.00
sellersEbayRelative_60	8487	5.59	7.69	0.00	44.00
priceDiff_60	8349	0.07	0.72	-5.08	6.66

Note: The table reports summary sample statistics for the merged scraped data from www.amazon.com and www.ebay.com. The last four variables *sellersEbayAll_60*, *sellersEbayRefined_60*, *sellersEbayRelative_60*, *priceDiff_60* are defined on a dataset constructed by searching only the first 60 characters of Amazon product title in eBay's search engine. They are used in robustness checks. Finally, we calculate the statistics of each variable with all valid observations in the dataset.

Table 2 presents summary statistics for our main variables of interest. For 32% of the products in our sample, Amazon acts as a middleman; for the other 68% products, Amazon acts as a pure platform. On average, the products have been on sale at Amazon for almost 5 years, although this varies across products from several days to 18 years. There is a large variation in the price and ranking. The maximum price is as high as $2118.83. The mean price is $64, the 25th percentile is $18.9, the 75th percentile is $72.9. The number of third-party sellers for a product ranges from 0 to 77 with a mean of 3.7 sellers. The number of sellers on eBay is much larger with a mean of 14.69 for *sellersEbayAll* and a mean of 6.53 for *sellersEbayRefined*. On average, the number of sellers on eBay is more than three times as high as the number of sellers on Amazon. Finally, variables with suffix 60 come from another dataset constructed for robustness checks and will be discussed later.

Table 3: Correlations among proxy variables

	logRank	*priceDiff*	*sellerEbay Refined*	*sellerEbay All*	*sellerEbay Relative*
logRank	1.0000				
priceDiff	-0.1359	1.0000			
sellersEbay Refined	-0.1462	-0.0901	1.0000		
sellersEbay All	-0.0595	-0.0797	0.7063	1.0000	
sellersEbay Relative	0.06839	-0.1203	0.6790	0.4866	1.0000

In table 3, the linear correlations among proxies are very weak. Correlations among proxies for different parameters are around 0.1.

Robustness Checks. We shall pursue a number of robustness checks. A first concern is that our result could be driven by the way that we count the number of eBay offers. To address this issue, instead of refining the list of eBay offers, we use the raw list of eBay offers to calculate the number of sellers, *sellersEbayAll*, and replace *sellersEbayRelative* by *sellersEbayAll*/*sellersAmazon*. Our results are robust to this change as shown in Table 4: although the coefficients of *sellersEbayAll* and *sellersEbayRelative* become smaller, they remain negative. The coefficient of *sellersEbayAll* now becomes non-significant, while *sellersEbayRelative* is still statistically significant. Relative to the result summarized in Table 1, the coefficients of the other variables remain almost the same.

Table 4: Regressions for Amazon's intermediation mode using the raw eBay search results

	(1) Linear	(2) Linear	(3) Probit	(4) Probit
sellersEbayRelative	-0.00354***		-0.00478***	
	(0.000428)		(0.000613)	
sellersEbayAll		-0.000226		-0.000439
		(0.000382)		(0.000426)
sellersAmazon		-0.000691		-0.000765
		(0.000915)		(0.000996)
log(rank)	-0.101***	-0.106***	-0.102***	-0.108***
	(0.00444)	(0.00461)	(0.00500)	(0.00513)
priceDiff	0.100***	0.112***	0.122***	0.136***
	(0.00844)	(0.00847)	(0.0111)	(0.0112)
log(price)	0.0346***	0.0402***	0.0440***	0.0503***
	(0.00614)	(0.00622)	(0.00689)	(0.00698)
listedDays	0.0606***	0.0664***	0.0719***	0.0788***
	(0.00479)	(0.00485)	(0.00596)	(0.00604)
Observations	6457	6457	6457	6457
Adjusted R^2	0.135	0.129		

Note: This table reports the robustness check using the raw eBay search results reflected in *sellersEbayAll* and *sellersEbayRelative*. Except for this change, the specification is the same as before.

A second concern is a bias caused by using the eBay search engine. We find that the number of offers provided by the eBay search engine is negatively correlated with the length of search text. In general, the longer the search text is, the lower the number of results that the eBay search engine can provide. Hence, the longer the product name is, the less likely it can find good matches in its database. This implies that we may ignore good matches if we provide a very long product name with too much information. For example, the same product may have different product titles by different sellers emphasizing different product features, such as size and color of the pan. In some cases, eBay can not give any offer when searching the whole Amazon product title, but does give the right offers when searching part of the Amazon product title. More importantly, there exists anecdotal evidence showing that the product title on Amazon is longer if it is registered by Amazon itself rather than by third-party sellers. If this is true, we may have spurious correlations. To solve this issue, we construct a second dataset by searching all product names using only the first 60 characters on eBay.[25]

The variables *sellersEbayAll_60*, *sellersEbayRefined_60*, *sellersEbayRelative_60* and *priceDiff_60* are constructed in this new dataset. As shown in the last four rows in the summary statistics Table 2, the average number of sellers for each product becomes larger. For example, in terms of the length of the raw search list, the average increases from 14.69 to 20.88. However, the relative prices between eBay and Amazon do not change much. The results using this new dataset are reported in Table 5 and yield similar relationships as our main ones. There are more observations in the regressions because some Amazon product titles which had no eBay offer before can now be matched. As in our previous regressions, the coefficients of other variables remain almost unchanged.

Table 5: Regressions for Amazon's intermediation mode using first 60 characters to search eBay offers

	(1) Linear	(2) Linear	(3) Probit	(4) Probit
sellersEbayRelative	-0.00450***		-0.00488***	
	(0.000595)		(0.000816)	
sellersEbayRefined		-0.000352		-0.000170
		(0.000499)		(0.000556)
sellersAmazon		-0.00587***		-0.00708***
		(0.000847)		(0.00112)
log(rank)	-0.101***	-0.0950***	-0.102***	-0.0984***
	(0.00423)	(0.00460)	(0.00468)	(0.00515)
priceDiff	0.114***	0.126***	0.134***	0.146***
	(0.00819)	(0.00849)	(0.0105)	(0.0106)
log(price)	0.0431***	0.0503***	0.0529***	0.0586***
	(0.00602)	(0.00614)	(0.00658)	(0.00667)
listedDays	0.0611***	0.0646***	0.0741***	0.0747***
	(0.00449)	(0.00480)	(0.00566)	(0.00576)
Observations	6822	6822	6822	6822
Adjusted R^2	0.138	0.100		

Note: This table reports the robustness check based on eBay search results using only the first 60 characters of the Amazon product title. Except for using new variable reflecting this change, *sellersEbayAll_60*, *sellersEbayRefined_60*, *sellersEbayRelative_60* and *priceDiff_60*, the specification remains the same as before.

[25]In our data, the median length of product title is 65, the minimum is 9, and the maximum is 245. We also tried to cut the first 50 or 80 characters. The results are similar.

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